MTP-AERO-62-21
March 1, 1962

N63 20191

Code-1

GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

STATUS REPORT #1
on
THEORY OF SPACE FLIGHT AND ADAPTIVE GUIDANCE

OTS PRICE

XEROX $ 8.10 ph

MICROFILM $ 2.72 mf

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MSFC - Form 523 (Rev. November 1960)

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2. For three disciplines, an introduction into the theory and a description of the present state of development are to be given. These disciplines are: celestial mechanics, calculus of variations, and the area of exploitation of large-scale computers.

3. The in-house and out-of-house (contracted) efforts for furthering our present day knowledge of the involved disciplines are to be discussed.

4. Results are to be presented that show the implementation of the theoretical achievements in the guidance mechanics applied to the Saturn vehicle flights.

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AEROBALLISTICS DIVISION

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SUMMARY

This volume is a collection of speeches given at the "Review Conference concerning Space Flight and Guidance Theory" held between various divisions of the Marshall Space Flight Center on February 20, 1962. The conference had the following objectives:

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INTRODUCTION

A Functional Perspective of Problems of Space Flight and Guidance Theory

by
Robert Silber

The following material is intended to present a kind of summary of the efforts now being expended in and under the supervision of the Future Projects Branch of the Aeroballistics Division in the area of Aeroballistics research, with especial emphasis on the adaptive guidance mode.

One need have only a superficial familiarity with the concept of the adaptive guidance mode to realize that any investigation involving the application or extension of the theory must necessarily entail almost every known discipline involved in the aeroballistics field. Of course, the same could probably be said for any guidance mode, since guidance ultimately determines the trajectory history of a given flight. Adaptive guidance, however, is broader in that it is conceptually capable of a continuous redetermination of, in some sense, a best trajectory history as a function of (a) state variables, (b) performance parameters and (c) mission criteria.

Because of the complexity of the investigations surrounding adaptive guidance, any presentation of the results of these investigations, if it is to be integrated, is in itself a problem. Some thought has therefore been devoted to the formulation of a structure within which particular results assume a relation to a total, rather than appear as a collection of seemingly disjointed and unrelated facts.

Such a structure is shown by Figure 1. This outline should enable the reader to perceive the role of a particular investigation in the overall division function. The diagram has two inputs, Missions and Aeroballistics Area. These are the two factors which determine the nature of any investigation undertaken within the Future Projects Branch. A certain "mission," in the broad sense, originates, say, in our Washington Headquarters. At this stage, the mission statement may not be mathematical, but is more likely to be stated in terms such as: to go around the moon and return with a specified vehicle in such a way that certain functions (such as photography) can be performed. Such missions can generally be construed to fall within one of the four broad categories listed in the diagram.

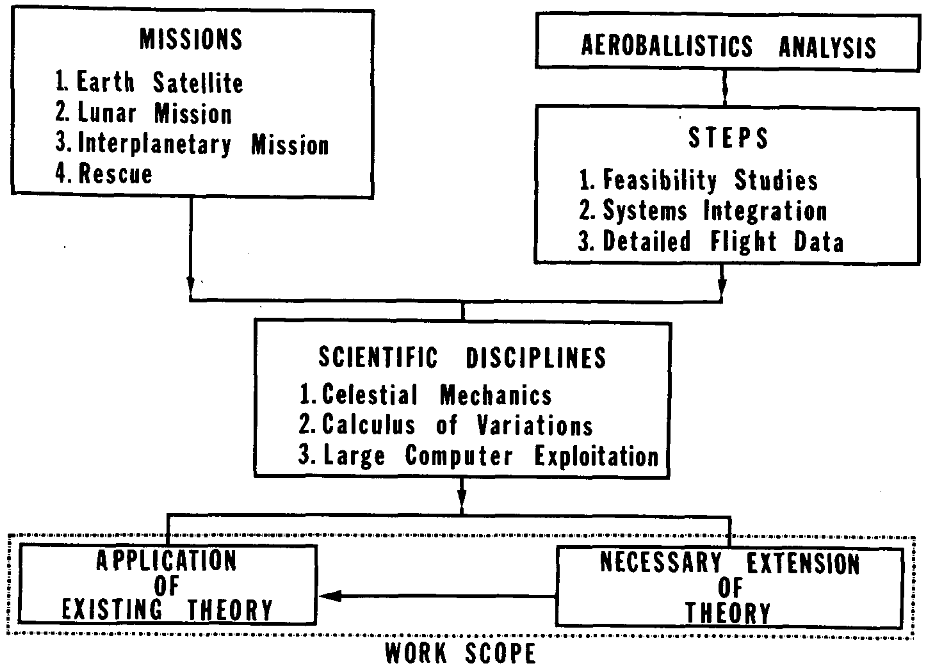

FIGURE 1

For each mission, a certain portion of the necessary analysis may be considered to fall within the realm of aeroballistics analysis, though not necessarily exclusively so. It has been found by past experience that the analysis of missions can be carried out roughly in three steps, shown in the diagram. The first of these, <u>feasibility studies</u>, concerns the very first rough analysis to determine whether the combination of vehicle and mission is feasible or marginal. The mission is then in some natural way divided into steps, and the physical phenomena of each are analyzed as to how they may be best performed separately and as to the problems they pose. In the second step, <u>systems integration</u>, the pieces must be fitted together to give the total picture, and a best overall solution extracted. This necessitates tradeoffs (from the ideal) in each step to achieve the most desirable integrated approach. Finally, for the actual execution of the mission, <u>detailed flight performance data</u> must be generated.

In order to carry out the investigation of the proposed missions, a certain body of <u>Scientific disciplines</u> is necessary. The nature of these disciplines, which is intended to be pointed out in Figure 1, is dependent upon the mission considered and the analysis to be performed. Listed in the figure are three of the advanced techniques considered necessary for development of the adaptive guidance mode. These are: Celestial Mechanics, Calculus of Variations, and Large Computer Exploitation.

The general work scope of the Future Projects Branch may then be defined as the (direct or indirect) necessary extension of existing theory and application of theory in the three steps previously described.

This, then, is the structure within which the descriptions that follow should be viewed. As each discipline and its application is described, it will be attempted to show its role relative to the total branch function and the total development of the adaptive guidance mode. Any meaningful evaluation as to the necessity of, applicability of and/or the significance of each investigation must be based on this more comprehensive perspective.

CELESTIAL MECHANICS - GENERAL PERTURBATIONS

By

M. C. Davidson

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CELESTIAL MECHANICS - GENERAL PERTURBATIONS

By

M. C. Davidson

SECTION I. INTRODUCTION

The problem of mission criteria formulation associated with the adaptive guidance mode becomes difficult when the mission involves a flight that is strongly affected by other bodies in the solar system in addition to Earth. The problem is to know what relationships must be satisfied by the coördinates of a vehicle at the termination of powered flight in order to produce a trajectory satisfying the mission. Present techniques involve guessing a set of coördinate values at the thrust termination point and stepwise computing the resulting trajectory. This guess is then successively improved until the trajectory satisfies mission requirements. This process of course does not answer the needs of the adaptive guidance mode. The desired relationships first mentioned do, however, and are being investigated now both in-house and by contractors under the contract, "Guidance and Space Flight Theory."

The problems encountered fall naturally into the field of celestial mechanics, and in particular, they are usually contained in the restricted n-body problem. The term restricted is used when it is understood that the motion of the mass under investigation, for example, a spacecraft, does not affect in any way the motion of the remaining masses.

Celestial mechanics is one of the most highly developed branches of mathematics. Euler, some two hundred years ago, considered many important problems in the field, (for example, the problem of two fixed centers and the restricted three body problem). The nineteenth century produced mathematicians such as Lagrange, Gauss, Legendre, Weierstrass, Jacobi, and Poincaré who contributed greatly to the field. The body of knowledge continues to grow in the twentieth century by the investigation of Wintner, Levi-Civita, Birkhoff, and Siegel, to mention a few. These mathematicians and others have produced techniques, which make up a large part of mathematical analysis, designed to solve problems of celestial mechanics. Even with such concentrated efforts the general motion in the n-body problem ($n > 2$) is not completely understood. The following is a discussion of some of the techniques applied to the problems of space travel.

For convenience the discussion of solutions and methods of solution is divided into two types, rigorous analytic solutions and perturbation techniques. Rigorous analytic solutions are taken to be those solutions which are mathematically exact in the sense that the variables may be computed to any desired degree of accuracy. The term "perturbation" represents the idea of considering the solution as a deviation from a known functional. Thus, we say the known functional is perturbed into the solution of the problem under investigation. The functional is usually the solution of a closely associated problem. A further division is made in perturbation methods as to their use. A general perturbation is understood to mean one which yields an approximation to the solution as an explicit function, valid for some class of the general solution. In other words, general perturbations are designed to provide information in the large about some class of orbits. In contrast to general perturbations, special perturbations are designed to facilitate the computation of orbits for a specific set of initial conditions. Such methods usually employ stepwise integration techniques.

Problems contained under the general headings, (rigorous analytic solution, general perturbations, and special perturbations), are discussed in that order.

SECTION II. DISCUSSIONS

A. RIGOROUS ANALYTIC SOLUTIONS

1. Euler's Problem of Two Fixed Centers

This problem consists of describing the motion of a point mass, P_3, under the influence of two other mass points, P_1 and P_2, which are fixed in space for all time. After the proper choice of length and mass, the Lagrangian function of P_3 in a space fixed cartesian system (x_1, x_2) is

$$L = \frac{1}{2}|\dot{x}|^2 + \frac{1-\mu}{|x+\mu|} + \frac{\mu}{|x+\mu-1|},$$

where $x = x_1 + i\,x_2$, $1-\mu$ and μ are the masses of P_1 and P_2 respectively and $|x|$ means the absolute value of x.

Let us make the transformation,

$$x_1 = \tfrac{1}{2}(\eta\ \xi + \sigma), \quad x_2^2 = \tfrac{1}{4}(\eta^2 - 1)(1-\xi^2), \text{ with}$$

$$\sigma = 1 - 2\mu\ .$$

We find the Lagrangian function L takes the form of Liouville and the system is soluble by quadratures.

The solution is expressed by the integral equations,

$$s = \sqrt{2} \int_{t_0}^{t} \frac{dt}{\eta^2 - \xi^2}$$

and
$$s = \int_{\eta_0}^{\eta} \frac{d\eta}{\sqrt{(\eta^2-1)(h\eta^2+8\eta+h_1)}} =$$

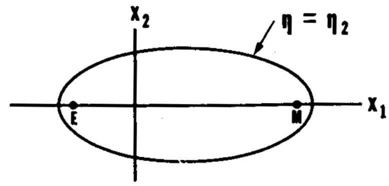
$$\int_{\xi_0}^{\xi} \frac{d\xi}{\sqrt{(1-\xi^2)(h\xi^2-8\sigma\xi+h_1)}}\ .$$

The total energy h and h_1 are constants of integration. We see that the introduced parameter, s, may be considered the new time variable. This follows from the fact that $\eta^2 - \xi^2$ is the product of the distances P_1P_3 and P_2P_3 and hence a non-negative function. We say s is a real increasing function of the real time, t.

The integrals in the solution are elliptic integrals; hence, η and ξ are elliptic functions of s. It can be shown that t is an elliptic integral of the third kind in the coordinates η and ξ. In order to represent the coordinates as explicit functions of s, it is necessary to make three distinctions of orbits. They will be called Class I, Class II, and Class III and may be illustrated as follows.

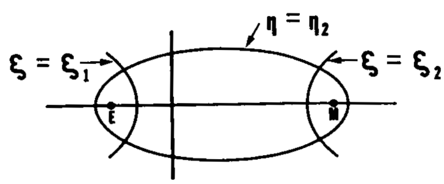
Class I represents orbits which will be contained for all time within a single ellipse, $\eta = \eta_2$.

CLASS I



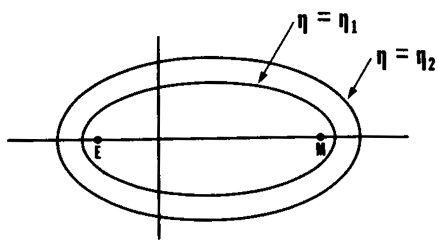
Class II represents orbits which are contained within a similar ellipse but are also constrained to lie to the left of the hyperbola, $\xi = \xi_1$, or to the right of the hyperbola, $\xi = \xi_2$, depending upon the initial point.

CLASS II



Class III represents orbits which are contained within the ellipse, $\eta = \eta_2$, but must lie outside the inner ellipse, $\eta = \eta_1$. The circumlunar orbits will be contained in Class I.

CLASS III



The explicit functional representation of Class I is

$$\eta = \frac{C_1 + C_2 \operatorname{sn}^2 \omega_1 (s + \phi_1)}{C_3 + \operatorname{sn}^2 \omega_1 (s + \phi_1)}$$

$$\xi = \frac{D_1 \operatorname{dn} \omega_2 (s + \phi_2) + D_2 \operatorname{sn} \omega_2 (s + \phi_2)}{D_3 \operatorname{dn} \omega_2 (s + \phi_2) + \operatorname{sn} \omega_2 (s + \phi_2)}$$

We see that the problem of two fixed centers can be solved exactly although the representation of the solution is different for Class I, II, and III. The solution of this problem in three dimensions follows exactly the same line as that in two dimensions. The importance of this problem is realized when one considers the possibility of perturbing these solutions into the solution of the restricted three body problem.

2. Power Series Representation to the Solution of the Three Body Problem

Sundman has shown that the equations of motion for the three body problem can be regularized; hence, they are soluble by power series. The time interval for which these series are valid may be chosen as any desired value. Dr. Schulz-Arenstorff of Computation Division has shown that this also applies to the restricted three body problem (Reference 1). Omitting the details of the method the solution has the form

$$x = \sum_{n=0} a_n \omega^h , \quad y = \sum_{n=0} b_n \omega^h, \text{ and } t = \sum_{n=1} c_n \omega^h$$

The importance of these series is that they in theory solve the three body problem; however, power series solutions offer no information in the large on the geometrical behavior of the orbits. Also, the series is not practical to use in the computation of orbits under present day machine capabilities due to their extremely slow rate of convergence.

3. Periodic Orbits in the Planar Three Body Problem

We wish to outline the construction of a family of periodic orbits in the restricted three body problem; however, it should be noted that the method is applicable to the planar three body problem with no restrictions on the masses. The existence proof and method of construction is due to Siegel (Reference 2).

In the restricted problem we are to construct periodic orbits of the infinitesimal point mass, P_1, about the mass point P_2 (of mass μ) where the mass points P_2 (of mass $1 - \mu$) and P_1 revolve in plane circles about their common center of mass. After the proper choice of the unit length, mass, and time, the differential equation of motion of P_3 in the usual rotating cartesian coordinate system (z_1, z_2) is

$$\ddot{z} + 2\,i\,\dot{z} - z = -\,(1-\mu)\,\frac{z+\mu}{|z+\mu|^3} - \mu\,\frac{z+\mu-1}{|z+\mu-1|^3}$$

where $z = z_1 + i\,z_2$.

Let us transform this equation to a coordinate system whose origin coincides with the point mass P_1. The desired transformation is simply

$$z + x = 1 - \mu$$

where $x = x_1 + i\,x_2$. The resulting equation is

$$\ddot{x} + 2\,i\,\dot{x} - x = (\mu-1)\left[1-(1-x)^{-\frac{1}{2}}\,(1-\bar{x})^{-\frac{3}{2}}\right] - \mu\,x^{-\frac{1}{2}}\,\bar{x}^{-\frac{3}{2}}$$

where $\bar{x}$ is the complex conjugate of x.

If the term $(\mu-1)\left[1 - (1 - x)^{-\frac{1}{2}}\,(1 - \bar{x})^{-\frac{3}{2}}\right]$

is expanded in a power series in x and $\bar{x}$ this series will converge absolute for $|x| < 1$. Grouping the linear terms we have

$$\ddot{x} + 2\,i\,\dot{x} + \tfrac{1}{2}(\mu-3)x + 3/2(\mu-1)\bar{x} + \mu\,x^{-\frac{1}{2}}\,\bar{x}^{-\frac{3}{2}} = P(x,\,\bar{x})$$

where P is a power series in x and $\bar{x}$ starting with second order terms.

Now consider the solution to the first order differential equations

$$\dot{\xi} = \alpha\,\xi\,, \qquad \dot{\eta} = -\,\alpha\,\eta$$

with $\alpha = \pm\frac{1}{4}\,i\,(\xi\eta)^{-3}$ and $\eta = \bar{\xi}$, the complex conjugate of ξ. It follows immediately that the product $\xi\eta$ is constant and further the solution is circular orbits in the $(\xi_1,\ \xi_2)$ rectangular coordinate system with $\xi = \xi_1 + i\,\xi_2$.

Let us attempt to represent the solution of the differential equations in x as a function of ξ and η. Further, let this solution take the form

$$x = \mu^{\frac{1}{3}} (1 \mp 2\zeta_3)^{\frac{1}{3}} \xi^4 (1 + \sum_{k=5} a_{k\ell} \zeta_{k\ell})$$

where

$$\zeta_k = \xi^{k+2\ell} \eta^{k-2\ell}$$

and ℓ may take on integers such that $2|\ell| \leq k$.

The existence proof reduces to showing the solubility for $a_{k\ell}$, ($2|\ell| \leq k$, $k = 1, 2, 3, \ldots$), and the convergence of the series for some positive values of $|\xi|$. This is accomplished by Siegel.

This series converges quite rapidly and, hence, lends itself to the actual computation of such orbits.

B. GENERAL PERTURBATIONS

1. Hamilton-Jacobi Approach

The particular procedure being considered by Republic Aviation takes the solution of Euler's problem of two fixed centers as the base of the solution to the restricted three body problem. The treatment of Euler's problem is to be the formal Hamilton-Jacobi technique. Let the Hamiltonian for Euler's problem be

$$F = F(x, y, t)$$

where x is the position defining vector and y is the momentum vector. Let us define a function s by the equation

$$F(x, s_x, t) + s_t = 0$$

where s_x is a vector whose components are $\frac{\partial s}{\partial x_i}$. Let the solution to this equation be $s = s(x, \alpha)$ where α is an introduced vector with constant components. The solution to Euler's problem is $x(t)$ defined by

$$s_\alpha = \beta$$

where β is a vector with constant components. The momentum $y(t)$ is given by

$$y = s_x.$$

The Hamiltonian for the restricted three body problem may be written as

$$H = F + G,$$

the Hamiltonian for Euler's problem plus the function G. Thus, the Hamilton-Jacobi equation becomes

$$H = F + G + s_t = G$$

since $F + s_t = 0$. The technique for approximating the solution of the reduced equation corresponding to the new Hamiltonian G is due to Delaunay.

2. Canonical Initial Conditions

The Hamiltonian for the restricted three body problem in the usual rotating cartesian coordinate system, (x_1, x_2) is

$$H = \tfrac{1}{2}(y_1^2 + y_2^2) - U + (x_2 y_1 - x_1 y_2)$$

where

$$U = \frac{1-\mu}{|x+\mu|} + \frac{\mu}{|x+\mu-1|}, \quad \text{and } x = x_1 + i\, x_2.$$

If we let

$$F = \tfrac{1}{2}(y_1^2 + y_2^2) - U$$

and

$$G = x_2 y_1 - x_1 y_2,$$

we have

$$H = F + G.$$

Under such a division of H, the function F is exactly the Hamiltonian function of Euler's problem of two fixed centers.

The differential equations of motion are produced by the matrix equation

$$\dot{z} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} H_{x_1} \\ H_{x_2} \\ H_{y_1} \\ H_{y_2} \end{bmatrix}$$

or we write simply

$$\dot{z} = J\ H_z.$$

Let the solution to the differential equations (of Euler's problem)

$$\dot{w} = J\ F_w$$

be $\quad w = w\ (w_o,\ t),$

functions of the initial conditions, w_o, and the time, t. Further, let us, within the solution $w = w\ (w_o,\ t)$, replace the initial conditions, w_o, by the unknown function of time ζ_k, $(k = 1,\ldots,4)$, and find the differential equation satisfied by ζ_k such that

$$z = w\ (\zeta(t),\ t).$$

The result is

$$\dot{\zeta} = J\ G^*_{\zeta}\ , \quad \text{where } G^* = G\ (w\ (\zeta,\ t)).$$

The process of dividing the Hamiltonian is again applied, producing an approximation for $\zeta_k(t)$ as

$$\zeta_1 = x_{20} \sin t + x_{10} \cos t$$

$$\zeta_2 = x_{20} \cos t - x_{10} \sin t$$

$$\zeta_3 = (\dot{x}_{20} + x_{10}) \sin t + (\dot{x}_{10} - x_{20}) \cos t$$

$$\zeta_4 = (\dot{x}_{20} + x_{10}) \cos t - (\dot{x}_{10} - x_{20}) \sin t$$

where $(x_{10}, x_{20}, \dot{x}_{10}, \dot{x}_{40})$ are the initial conditions in a rotating coordinate system.

Reference 3 treats the method in detail.

3. The Parameters of Mean Motion

Let us consider the system of differential equations

$$\dot{x}_k = f_k\,(x, \ldots, x\,, \omega)\,, \qquad (k = 1, \ldots, 4)$$

where the right hand sides are functions of the coordinates $(x_1, \ldots, x_4)$ and the parameter ω. The equations of motion for the restricted three body problem in a rotating coordinate system may be put into this form. The University of Kentucky is conducting a study where the parameter is the mean motion. Consider the solution of these differential equations to have the form

$$\dot{x}_k = \sum_{j=0} a_{kj}\,(\xi, t)\,\omega^j$$

which is a power series in ω with the coefficients as functions of the initial point, $x_k(t=0) = \xi_k$, and time. The differential equations for $a_{k,j}$ are

$$\frac{d}{dt}\,(a_{k,j}) = b_{k,j} \quad , \qquad (k = 1, \ldots, 4\ ,\ j = 1, 2, 3, \ldots)$$

where

$$f_k\ (x_1, \ldots, x_4,\ \omega) = \sum_{j=0} b_{kj}\ \omega^j \quad , \ (k=1, \ldots,\ 4).$$

Due to the fact that if the mean motion ω was zero, Euler's problem would be produced, the $a_{k,0}$ $(k=1, \ldots, 4)$ are the coordinates within the Euler problem.

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3. "Reduction of the Restricted Problem of Three Bodies to a Perturbation of Euler's Problem of Two Fixed Centers," Report No. MTP-AERO-61-62, by M. C. Davidson, Jr. and Dr. R. Schulz-Arenstorff.

CELESTIAL MECHANICS

SPECIAL PERTURBATIONS AND TWO-BODY PROBLEM

By

Hans Sperling

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SECTION I. INTRODUCTION

In the following a brief outline shall be given of the work that Mr. Tucker of Future Projects Branch and I are doing on the numerical computation of orbits. The main object of this work is to get economic decks of various degrees of accuracy for three types of trajectories: 1. trajectories around one body, especially the earth, 2. trajectories in the field of two bodies, especially the earth-moon space, and 3. trajectories in the field of several bodies, i.e., the solar system.

The N-Body Problem, except for N=2, cannot be solved in a convenient closed form, apart from a very few special cases. Thus one is more or less forced to use numerical integration for the actual computation of orbits, especially if the bodies cannot be considered to be point masses.

SECTION II. DISCUSSIONS

A. TYPES OF DECKS

Jet Propulsion Laboratory furnished Aeroballistics Division with an accurate and versatile interplanetary deck, which suffices for all present purposes and those coming up in the near future; thus there is no need at this time for the development of an own interplanetary deck. On the other hand, the decks which we are establishing now are suitable for the computation of interplanetary trajectories after only minor changes, although they would not be very convenient.

The computation of trajectories of earth satellites and that of lunar trajectories pose essentially the same problems; thus, there is no need to make any distinctions between these in the following.

B. SPECIAL PERTURBATION METHODS

The classical theory of special perturbations - the term "special perturbations" used for all methods of numerical integration here - offers mainly three methods for numerical work: Cowell's method, Encke's method, and the variation of parameters method. The first two are used almost exclusively for numerical work, while the equations of the third method are also a starting point for general perturbation theory, i.e., for analytical investigations. In addition, we are also investigating as a generalization of Encke's method the so-called Varicentric method, which has been developed in Future Projects Branch in order to overcome certain difficulties of the older methods.

Cowell's method is the straightforward numerical integration of the differential equations of motion, which are referred either to barycentric coordinates or to one of the bodies (m_o).

BARYCENTRIC COORDINATES

$$\ddot{\bar{r}} = -\sum_{\nu=0}^{N} \gamma m_\nu \frac{\bar{r} - \bar{r}_\nu}{|\bar{r} - \bar{r}_\nu|^3}$$

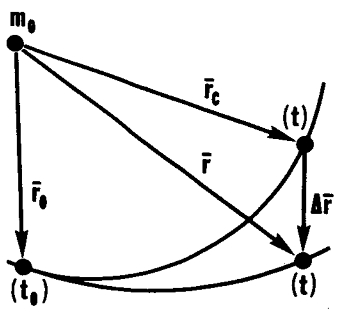
COORDINATES REFERRED TO MASS m_0

$$\ddot{\bar{r}} = -\gamma m_0 \frac{\bar{r}}{r^3} - \sum_{\nu=1}^{N} \gamma m_\nu \left(\frac{\bar{r} - \bar{r}_\nu}{|\bar{r} - \bar{r}_\nu|^3} + \frac{\bar{r}_\nu}{r_\nu^3} \right)$$

$$\bar{r}_\nu = \bar{r}_\nu(t)$$

This method poses the least problems, but it is also generally the least accurate or, referred to a fixed accuracy, the slowest of the methods mentioned here. It should be mentioned that in the astronomical literature the name "Cowell's method" is mostly connected with a specific integration scheme, while the definition given here is that commonly used in astronautics.

Encke's method is applicable when the space ship moves in the central field of one celestial body (m_0) so that the influence of the other bodies can be considered as a small perturbation. The trajectory $\bar{r}$ is approximated piecewise by conic sections $\bar{r}_c$, which are determined by the central field of m_0 and by having at a certain time t_0 the same initial values as the true trajectory:



ENCKE'S METHOD

$$\bar{r}_0 = \bar{r}(t_0) = \bar{r}_c(t_0) \qquad \dot{\bar{r}}_0 = \dot{\bar{r}}(t_0) = \dot{\bar{r}}_c(t_0)$$

The difference $\Delta\bar{r}$ between the true and the approximate position, $\bar{r} = \bar{r}_c + \Delta\bar{r}$

is numerically integrated. Introducing

$$\ddot{\bar{r}}_c = -\gamma m_0 \frac{\bar{r}_c}{r_c^3}$$

into the differential equations of motion referred to m_0, one gets

$$\Delta\ddot{\bar{r}} = -\gamma m_0 \left(\frac{\bar{r}}{r^3} - \frac{\bar{r}_c}{r_c^3} \right) - \sum_{\nu=1}^{N} \gamma m_\nu \left(\frac{\bar{r} - \bar{r}_\nu}{|\bar{r} - \bar{r}_\nu|^3} + \frac{\bar{r}_\nu}{r_\nu^3} \right)$$

The main advantages are: 1. the difference $\Delta\bar{r}$ is usually much smaller than the true coordinate $\bar{r}$ itself, so that the numerical integration can be performed with less accuracy and therefore faster; 2. certain terms in the differential equations, which appear as the difference of two almost equal expressions, can be expanded into fast converging series, thus increasing accuracy by analytically removing the large parts of the term and increasing speed of computation by simplifying the term.

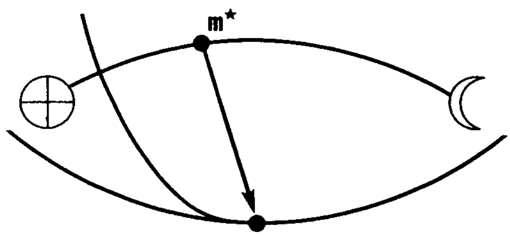
Series expansion with $(1 + u)^{-3/2} = 1 + B(u)$

$$\frac{\bar{r}}{r^3} - \frac{\bar{r}_c}{r_c^3} = \frac{1}{r_c^3}\left(\Delta\bar{r} + \bar{r}B(X_c)\right) \qquad X_c = \frac{2}{r_c^2}\left((\bar{r}_c\Delta\bar{r}) + (\Delta\bar{r})^2\right)$$

$$\frac{\bar{r} - \bar{r}_\nu}{|\bar{r} - \bar{r}_\nu|^3} + \frac{\bar{r}_\nu}{r_\nu^3} = \frac{1}{r_\nu^3}\left(\bar{r} + (\bar{r} - \bar{r}_\nu)B(X_\nu)\right) \qquad X_\nu = -\frac{2}{r_\nu^2}\left((\bar{r}\,\bar{r}_\nu) + r^2\right)$$

Varicentric method This is a generalization of Encke's method insofar as the true trajectory is again approximated by a conic section; but now the conic section is not referred to one of the real bodies, as earth or moon or sun, but to a fictitious body with both variable mass and position. The fictitious body m* is chosen so that its central field approximates the real field in the neighborhood of the space ship "as well as possible". In addition, this fictitious body shall coincide with the real body in the special case of a central field, and it shall be very close - in mass and position - to a real body if the space ship moves very close to this real body. This method is intended to avoid the difficulty of changing the reference body, which is necessary in Encke's method and the variation of parameters method, if the space ship moves from the neighborhood (i.e., the central field) of one body to that of another. In the varicentric method the fictitious reference body moves, generally continuously, from the one real body to the other one.

VARICENTRIC METHOD



Variation of parameters method The differential equations of motion

$$\ddot{\bar{r}} = -\gamma m_0 \frac{\bar{r}}{r^3} + \bar{F} \qquad \bar{F} = -\mathrm{grad}_{\bar{r}} R$$

$$R = -\sum_{\nu=1}^{N} \gamma m_\nu \left(\frac{1}{|\bar{r} - \bar{r}_\nu|} + \frac{(\bar{r}\,\bar{r}_\nu)}{r_\nu^3}\right)$$

are transformed to a new set of dependent variables:

New Variables $\xi_1,\ldots,\xi_6$ by

$$\bar{r} = \bar{f}(\xi_1,\ldots,\xi_6,t) \qquad \dot{\bar{r}} = \bar{g}(\xi_1,\ldots,\xi_6,t)$$

New Differential Equations

$$\dot{\xi}_j = h_j(\xi_1,\ldots,\xi_6,t) \qquad j = 1,\ldots,6$$

These new variables are usually chosen to be constants of the Two-Body motion; i.e., for the case of the Two-Body problem the solution of the new differential equations reduces to constants.

Special Set of New Variables : Keplerian Elements

The transformation equations are the equations of the two-body problem.

a = semimajor axis	ε = mean longitude at epoch $t=0$
e = eccentricity	mean longitude $\lambda = nt + \varepsilon$
I = inclination	$\tilde{\omega}$ = longitude of perihelion
	Ω = longitude of ascending node

With these variables the new set of differential equations reads:

$$\dot{a} = \frac{2}{na}\frac{\partial R}{\partial \varepsilon}$$

$$\dot{e} = -\frac{\sqrt{1-e^2}\,(1-\sqrt{1-e^2})}{na^2 e}\frac{\partial R}{\partial \varepsilon} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \tilde{\omega}}$$

$$\dot{I} = -\frac{\tan\frac{1}{2}I}{na^2\sqrt{1-e^2}}\left(\frac{\partial R}{\partial \varepsilon} + \frac{\partial R}{\partial \tilde{\omega}}\right) - \frac{1}{na^2\sqrt{1-e^2}\,\sin I}\frac{\partial R}{\partial \Omega}$$

$$\dot{\varepsilon} = -\frac{2}{na}\frac{\partial R}{\partial a} + \frac{\sqrt{1-e^2}\,(1-\sqrt{1-e^2})}{na^2 e}\frac{\partial R}{\partial e} + \frac{\tan\frac{1}{2}I}{na^2\sqrt{1-e^2}}\frac{\partial R}{\partial I}$$

$$\dot{\tilde{\omega}} = \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} + \frac{\tan\frac{1}{2}I}{na^2\sqrt{1-e^2}}\frac{\partial R}{\partial I}$$

$$\dot{\Omega} = \frac{1}{na^2\sqrt{1-e^2}\,\sin I}\frac{\partial R}{\partial I}$$

C. PERTURBATIVE TERMS

There are no principal difficulties in establishing the equations of motion, taking into account all major gravitational influences. The practical question concerning the evaluation of these equations arises: Which terms of the equations can be simplified or neglected in order to speed up computation, without giving up the desired accuracy? In our case of trajectories in the neighborhood of the earth or in the earth-moon space, we have essentially two sources of gravitational perturbations: first, distant celestial bodies, as sun, planets, and moon or earth (e.g., in Encke's method the moon (earth) will perturb the motion, if the earth (moon) is the reference body); secondly, the oblateness of the earth or moon.

Oblateness Consider first the perturbations caused by the oblateness of the earth (the same considerations hold also for the moon). For practical reasons they are divided into three types: "coordinate system", "precession and nutation", and "equatorial bulge". They shall be discussed briefly.

Coordinate system Physically, of course, the choice of the coordinate system is not considered to introduce perturbations; but in a formal-mathematical sense, the change to a new coordinate system can introduce additional terms into the differential equations, which can be considered formally as perturbations. In this sense the notation should be understood.

The plane of the mean equator of date moves relative to that of the mean equator of a fixed epoch, the motion being a rotation through about 20" per year. The tabulated coordinates of the celestial bodies are usually referred to the equatorial (or ecliptical) coordinate system of a fixed epoch, mainly 1950.0, or to the mean equator (ecliptic) of date. Assume that we want to compute a trajectory for, say 1970. At this time the mean equator of date, which is the reference plane for the oblateness terms, has an inclination of about 7' relative to that of 1950.0, and the question is: How much does this displacement of the equatorial plane influence the trajectory? Or, in other words: How much does a trajectory differ from the true one, if we disregard this motion of the equatorial plane? Rigorous, but time-consuming solutions are: 1. Compute in the coordinate system of 1950.0 and consider the oblateness terms as functions of time; or 2., formulate the equations of motion in an equatorial coordinate system of date, and transform the coordinates of the celestial bodies into this system.

Precession and nutation As mentioned, the equator of date moves relative to that of a fixed epoch. The secular part of this motion is called precession, the periodic or almost periodic part is called nutation. The question, similar to that just discussed, is: Of what type and magnitude is the error, if we neglect the motion of the equator and consider it fixed? How long can it be kept fixed without introducing an appreciable error, a few days or weeks or longer? The answer depends of course on the trajectory, i.e., how long it remains in the neighborhood of the earth. It should be mentioned that the nutation is small, namely, the nutation in longitude is less than 10", the nutation in obliquity less than 20".

Equatorial bulge The potential of an oblate body can be represented as a series of Legendre polynomials, and accordingly one distinguishes between first, second, third ... harmonics. The practical question arising here is: Which oblateness terms do influence the trajectory and up to which distance from the earth for a required accuracy of the computation?

Celestial bodies The question is quite similar: which bodies of the solar system (sun, planets) do influence the trajectory so much that they have to be taken into account and how large is this influence?

D. TWO-BODY PROBLEM

The Two-Body motion is a good approximation to the true motion in many cases of celestial mechanics. Furthermore, the solution of the Two-Body problem is the basis for various perturbation methods: For instance, in Encke's method a Keplerian conic section is used as an approximation to the true trajectory, and also the variation of parameters method is based on the solution of the Two-Body problem. Therefore a complete theory of the Two-Body problem, that is also satisfactory for numerical purposes, is important for applications.

The following simple, but not unrealistic example shows clearly that in certain limiting cases the classical formulas for the Two-Body motion are unsatisfactory for numerical computations.

Let the conic section be a near parabolic ellipse. Assume that

$$\gamma m = 1 \ ; \ r_o = \text{distance} = 2(1 - 10^{-7} \pm 10^{-8}) \ ; \ v_o = \text{velocity} = 1 \pm 10^{-8}$$

The semimajor axis a is given by $a = \dfrac{r_o}{2\left(1 - \dfrac{r_o v_o^2}{\gamma m}\right)}$

One easily finds that

$$r_o v_o^2 = 2(1 - 10^{-7} \pm 3.10^{-8})$$

and that

$$a \approx \frac{1}{10^{-7} \pm 3.10^{-8}} = 10^7 \frac{1}{1 \pm .3} = 10^7(1 + \Delta)$$

where

$$-.23 \leqslant \Delta \leqslant .43 \ ,$$

such that

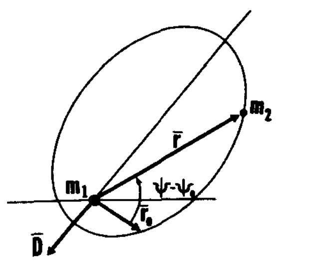
$$.77\ 10^7 \leqslant a \leqslant 1.43\ 10^7 .$$

Thus, only the order of magnitude of $\underline{a}$ can be derived from the given numbers, and it is obvious that the classical formulas

$$r = a(1 - \cos E)\ ;\ x = a(\cos E - e)\ ;\ y = a\sqrt{1 - e^2}\,\sin E$$

can yield only the order of magnitude of the coordinates, even in the neighborhood of r_o.

A brief description of a new set of formulas for the Two-Body motion follows.



$m = m_1 + m_2 \qquad r = |\bar r|$

TWO-BODY PROBLEM

The equations of motion for $\bar r$ and r read:

$$\ddot{\bar r} = -\gamma m \frac{\bar r}{r^3} \qquad \ddot r - \frac{2h - \dot r^2}{r} - \frac{\gamma m}{r^2} = 0$$

Introduce a new uniformizing variable s by

$$s - s_0 = \sqrt{\gamma m}\int_{t_0}^{t} \frac{dt}{r(t)} .$$

Then $\bar r$, r, and t become functions of s, and the new differential equations read, denoting the differentiation with respect to s by a prime,

$$\bar r'' + h^* \bar r + \bar D = 0 \qquad r'' + h^* r - 1 = 0 \qquad t' = \frac{1}{\sqrt{\gamma m}}\, r$$

where

$$|\bar D| = e \qquad h^* = -\frac{2h}{\gamma m} = \frac{1}{a}$$

The vector $\bar D$ is directed towards the pericenter of the conic section, if $\bar D \neq 0$.
Both the equations for $\bar r(s)$ and $r(s)$ are of the simple type

$$(*) \qquad w''(u) + M w(u) + N = 0$$

M and N being constants.
Introduce functions $S_j(u)$ by $\quad S_j(u) = \sum_{\nu=0}^{\infty} (-1)^\nu \frac{u^\nu}{(2\nu+j)!}$

Then, for instance,

$$S_0(u^2) = \cos u \qquad S_1(u^2) = \frac{\sin u}{u}$$

$$S_0(-u^2) = \cosh u \qquad S_1(-u^2) = \frac{\sinh u}{u}$$

The general solution of (*) can be written as

Initial Values $u_0,\ w_0,\ w_0'$

$$w = w_0 + w_0'(u - u_0)\, S_1\left[M(u-u_0)^2\right] - (N + w_0 M)(u-u_0)^2\, S_2\left[M(u-u_0)^2\right]$$

Applying this result to the differential equations for $\bar{r}(s)$ and $r(s)$, we get

$$\bar{r} = \bar{r}_0 + \frac{r_0 \dot{\bar{r}}_0}{\sqrt{\gamma m}} (s-s_0)\, S_1\left[h^*(s-s_0)^2\right] - (\bar{D} + \bar{r}_0 h^*)(s-s_0)^2\, S_2\left[h^*(s-s_0)^2\right]$$

$$r = r_0 + \frac{r_0 \dot{r}_0}{\sqrt{\gamma m}} (s-s_0)\, S_1\left[h^*(s-s_0)^2\right] + (1 - r_0 h^*)(s-s_0)^2\, S_2\left[h^*(s-s_0)^2\right]$$

By simple calculations one finds then general expressions for Kepler's and Gauss' Equations:

Kepler's Equation

$$\sqrt{\gamma m}\,(t - t_0) = r_0 (s-s_0) + \frac{r_0 \dot{r}_0}{\sqrt{\gamma m}} (s-s_0)^2 S_2\left[h^*(s-s_0)^2\right] + (1 - r_0 h^*)(s-s_0)^3 S_3\left[h^*(s-s_0)^2\right]$$

Gauss' Equation

$$\cot \tfrac{1}{2}(\psi - \psi_0) = \frac{2 r_0}{\sqrt{p}} \, \frac{S_0\left[\tfrac{1}{4} h^*(s-s_0)^2\right]}{(s-s_0)\, S_1\left[\tfrac{1}{4} h^*(s-s_0)^2\right]} + \frac{r_0 \dot{r}_0}{K}$$

SECTION III. APPLICATIONS

Various decks using the aforementioned methods have been established or are being established. The most important of these are listed in the following brief survey.

A. TRAJECTORY COMPUTATION DECKS

Deck for earth satellites

Purpose: investigations and applications to actual problems
3-dimensional -- includes oblateness and 3 perturbing point masses
Encke's method -- integration method: Nyström -- coded in single and double precision
Operational with: idealized coordinates of the perturbing bodies and eccentricity $e < 1$.

Decks for the Restricted Three-Body Problem

Purpose: investigations
2-dimensional -- integration method: Nyström -- coded in double precision

1.)	Cowell's method, geocentric coordinates	operational
2.)	Cowell's method, barycentric coordinates	operational
3.)	Encke's method, geocentric coordinates	operational
4.)	Encke's method, shifting reference body	operational
5.)	Varicentric method, geocentric coordinates	checkout
6.)	Varicentric method, barycentric coordinates	checkout

CALCULUS OF VARIATIONS

By

David Schmieder

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CALCULUS OF VARIATIONS

By

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SECTION I. INTRODUCTION

An attitude often found in Future Projects Branch is that difficult problems existing at this time in space flight and guidance theory should be attacked in their complete form and solved stepwise, rather than to be watered down and covered up with assumptions which enable the individual problems to be solved completely and immediately. Among the many advantages believed to be gained by this approach are: (1) A large carry-over of knowledge from the work done in connection with one specific application to the work to be done in connection with the next, since most applications are based on the same general theory. If many restrictions and simplifying assumptions are made for one application, the work done is not likely to give much information toward the next. (2) The "state of the art" in the development and use of the underlying scientific disciplines tends to be advanced, so as to aid in the solution of more difficult problems in the future, and (3) a better control is had over the problem, in that the degree to which the ideal solution of the physical problem is attained is known.

Such a philosophy has influenced the development of the adaptive guidance mode, as exemplified by the incorporation of optimum trajectories into the corresponding guidance processes. Also, by the nature of the scope of work of the branch as described earlier in this report, much trajectory computation is involved, with certain flight mechanical specifications to be met. Certainly, the most economical of such trajectories must be found, for even if they were not used in practice, they would be needed for comparison purposes. If this optimization problem is stated in complete form, then it has as its "independent variable" an entire function, the thrust direction function and, therefore, automatically falls into the realm of the calculus of variations.

SECTION II. DISCUSSIONS

A. GENERAL DESCRIPTION OF THE VARIATIONAL PROBLEM

The scientific discipline represented by the calculus of variations has been receiving increased attention recently, and still needs further development in many areas. To illustrate these areas, we may look at the breakdown of a general problem in the calculus of variations as shown in Figure 1.

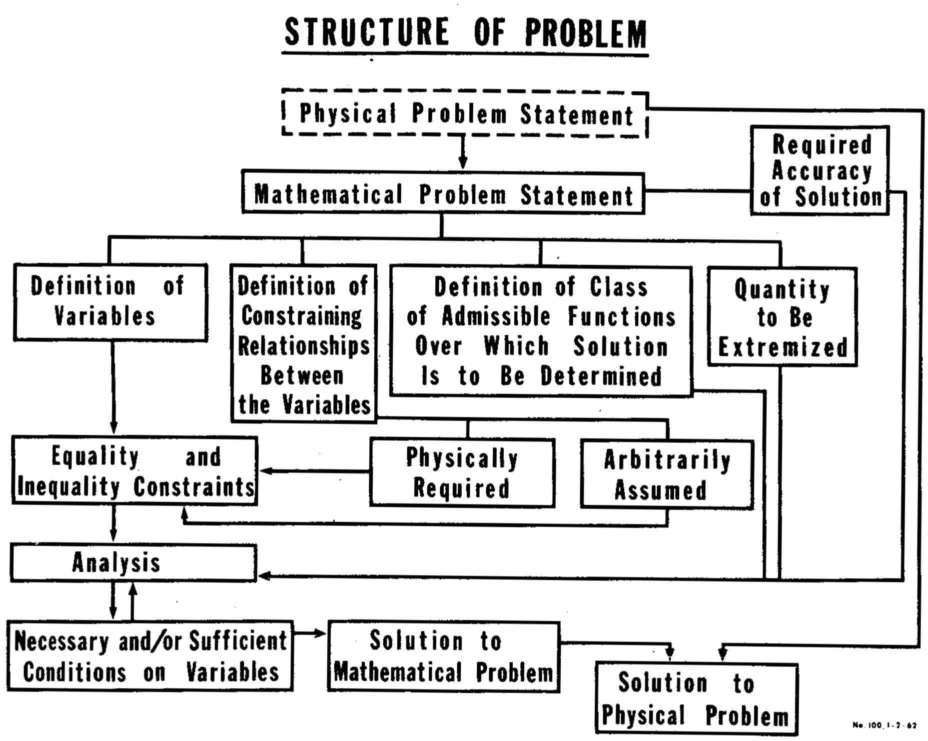

FIGURE 1

For the work in our branch, we usually have as a basis some physical statement of the problem. To arrive at the solution to this physical problem ordinarily requires a formulation of the problem in mathematical form. The solution of that mathematical problem should, with proper interpretation, determine the desired physical problem solution. This step in the solution is often a difficult one, and is a point where we wish to avoid making arbitrary simplifications.

A typical mathematical problem statement may be broken down into the basic parts shown in Figure 1. This includes first a definition of the variables in the problem, and therefore, an implication that if any other quantities vary, their variation does not affect the problem within the required accuracy of solution. Next, there are certain constraints on these variables, imposed by the physical situation, which must be formulated mathematically. Then, corresponding to the physical problem of best utilizing the remaining degrees of freedom for these variables, there is a mathematical statement defining a quantity to be extremized and the class of functions which are to be considered in the search for a solution.

Note that the constraining relationships are further broken down into those physically required and those arbitrarily assumed. The same may be said of the class of admissible functions. Such relationships that are arbitrarily assumed for the sake of expediency are what we wish to eliminate, so as to give as much freedom as possible to the extremizing functions. A great variety of equality and inequality constraints, and quantities to be extremized, result from the various physical problems that we face; and each presents its own particular difficulty in the mathematical analysis to be made. The result looked for from the analysis is a set of conditions on the variables which are both necessary and sufficient to meet the specified conditions of the problem, and which are in a useful form. At the present state of development, this result is only partially available.

In an effort to more closely approach this goal, several methods of attacking the variational problem are currently being pursued, and are listed in Figure 2.

This figure also shows the relation of the theory to the applications made in our work, and shows in what areas some of the contractors are working. The symbols referring to the various contractors are defined on page 83 of this report. These applications will be discussed after a general description of the general nature of the approaches shown in Figure 2 has been given.

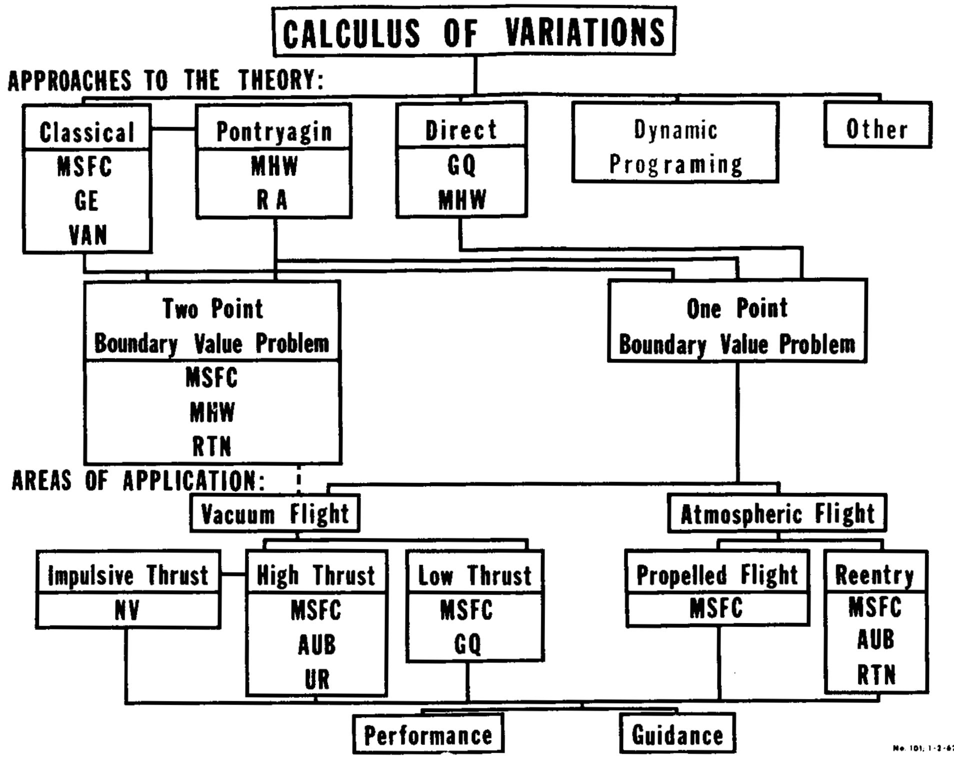

FIGURE 2

B. EXAMPLE PROBLEM

For an example with which to illustrate the various approaches, consider the flat earth problem as shown in Figure 3.

In order to more clearly show the main characteristics of the approaches and the analogies between them, a vector notation is used. The components x_3, x_4 are cartesian coordinates of the point assumed to have mass x_5, and x_1, x_2 their first time derivatives. Motion is influenced only by a constant gravitational acceleration $\bar{g}$ and a thrust vector having a magnitude F for time $t_0 < t < T$, zero for all other time, and which makes an angle χ with the x_4 axis. It is desired to find the time history $\chi(t)$ which causes conditions to be reached at $t = T$ such that setting

FLIGHT GEOMETRY FOR AN EXAMPLE PROBLEM

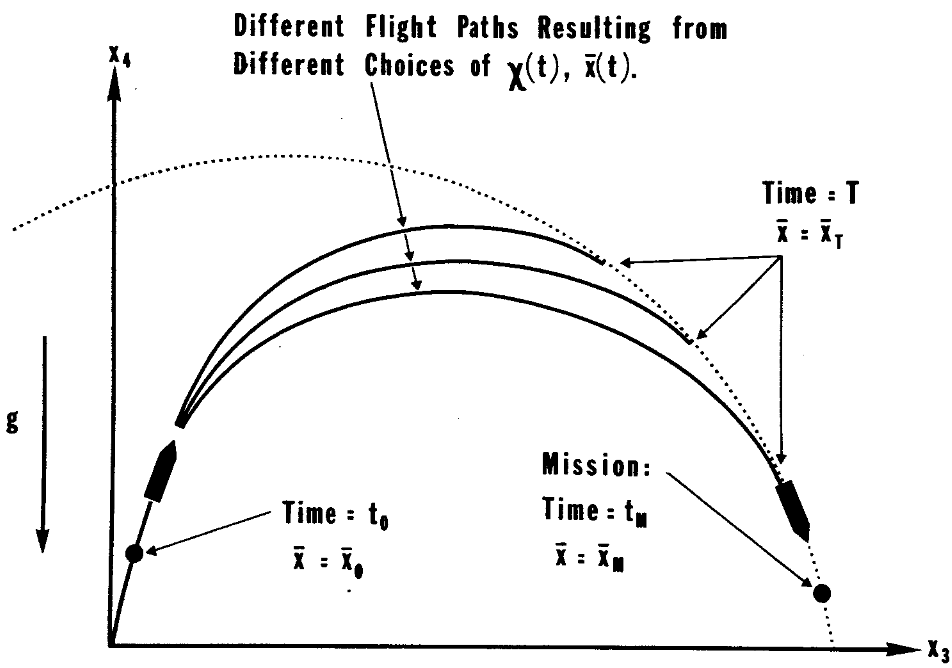

FIGURE 3

$F = \dot{x}_5 = 0$ for $t > T$ causes $\bar{x}(t_M) = \bar{x}_M$ at some given later time t_M, and in addition accomplishes this with the maximum value of $x_5(T)$. Newton's laws of motion are assumed.

The mathematical formulation for this problem is given in Figure 4. The constraints are given by the equation of motion (1.), the initial condition (2.) and the final conditions (3.). The quantity to be extremized and the class of admissible functions are given by (4.). For this example problem, no claim is made for applicability to any known physical problem, as it is over simplified for demonstration purposes.

MATHEMATICAL FORMULATION

1. Equations of Motion:

$$\dot{\bar{x}} = \begin{bmatrix} \frac{F}{x_5}\sin\chi \\ \frac{F}{x_5}\cos\chi - g \\ x_1 \\ x_2 \\ K_1 \end{bmatrix} \equiv \bar{f}$$

2. Initial Conditions:

$$\bar{x}(t_0) = \begin{bmatrix} x_{10} \\ x_{20} \\ x_{30} \\ x_{40} \\ K_2 \end{bmatrix}$$

3. Mission Equations:

$$\bar{x}_T = \begin{bmatrix} x_{1M} \\ x_{2M} - g(t_M - T) \\ x_{3M} - x_{1M}(t_M - T) \\ x_{4M} - x_{2M}(t_M - T) - \frac{1}{2}g(t_M - T)^2 \\ K_1 + K_2 T \end{bmatrix}$$

4. Mathematical Problem

To find among all [twice differentiable] functions

$$\bar{x}(t) \quad , \quad \chi(t)$$

that set which satisfies [1., 2., and 3.,] and maximizes $[x_5]$.

FIGURE 4

C. CLASSICAL APPROACH

An outline of the procedure taken by the classical approach to this problem is given in Figure 5. First, the vector $\bar{g}$ and scalar product G are defined as shown, where $\bar{f}$ is as defined in Figure 4. Thus, imposing the constraint $\bar{g}$ causes the equations of motion to be satisfied. A result of the classical theory then is the necessity for $\bar{\lambda}$ to exist such that the given Euler-Lagrange equations are satisfied. For the present example these are written out in equations (5.). Thus, it is seen that the problem of finding the optimum function becomes the problem of solving the two-point boundary value problem represented by the system of differential equations (5.) and (1.), together with part of the end conditions at the two-point t_0 and T, given by (2.) and (3.), respectively. What a solution would consist of may be defined in two slightly different forms. One would be the determination of the optimum functions $\bar{x}(t)$, $\chi(t)$,

CLASSICAL APPROACH

Define $\bar{g} = \dot{\bar{x}} - \bar{f}$

$G = \bar{\lambda} \cdot \bar{g}$

A necessary condition is that $\bar{\lambda}$ exists satisfying the Euler-Lagrange equations :

$$\overline{\frac{\partial G}{\partial x}} = \overline{\frac{\partial \dot{G}}{\partial \dot{x}}}$$

and

$$\frac{\partial G}{\partial \chi} = \frac{\partial \dot{G}}{\partial \dot{\chi}}$$

or

$$\begin{bmatrix} -\lambda_3 \\ -\lambda_4 \\ 0 \\ 0 \\ \frac{F}{x_5^2} (\sin\chi + \cos\chi) \end{bmatrix} = \dot{\bar{\lambda}}$$

and

$$\lambda_1 \cos\chi - \lambda_2 \sin\chi = 0 \qquad 5.$$

Solve the resulting two point boundary value problem: 1.,2.,3.,5.+ additional necessary conditions.

FIGURE 5

and T, as required for performance work. The other, more applicable to guidance work, would be the determination of the necessary values of the remaining initial conditions $\bar{\lambda}(t_o)$ and of T as functions of the given $\bar{x}(t_o)$ and mission constants in equations (3.).

The additional necessary conditions mentioned at the bottom of the chart refer to other necessary conditions found in the theory which are needed if sufficiency is to be demonstrated. These involve such things as distinguishing maxima from minima, and insuring proper behavior at the end points.

D. PONTRYAGIN APPROACH

The Pontryagin approach starts by defining the quantity to be extremized in the form of a scalar product of two vectors and defining the variable $\bar{p}$ through differential equations as shown in Figure 6. Then when H is defined as

PONTRYAGIN APPROACH

To maximize $S = \bar{c} \cdot \bar{x}(T)$ where $\bar{c} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$,

Define $\bar{p}$ by $\dot{\bar{p}}_i = -\bar{p} \cdot \bar{f}_{xi}$, $\bar{p}(T) = -\bar{c}$

$$\text{or} \quad \dot{\bar{p}} = \begin{bmatrix} -p_3 \\ -p_4 \\ 0 \\ 0 \\ \frac{F}{x_5^2}(p_1 \sin\chi + p_2 \cos\chi) \end{bmatrix}, \quad \bar{p}(T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \qquad 6.$$

so that if H is defined by $H = \bar{p} \cdot \bar{f}$

then $\dot{\bar{x}} = \bar{H}_p$ and $\dot{\bar{p}} = -\bar{H}_x$.

The Maximum Principle states that H must be a maximum with respect to χ at every time t :

$$\frac{\partial H}{\partial \chi} = 0 \qquad \frac{F}{x_5}(p_1 \cos\chi - p_2 \sin\chi) = 0 \qquad 7.$$

$$\frac{\partial^2 H}{\partial \chi^2} < 0 \qquad -\frac{F}{x_5}(p_1 \sin\chi - p_2 \cos\chi) < 0 \qquad 8.$$

Solve the resulting two point boundary value problem : 1., 2., 3., 6., 7., under restriction 8. plus additional necessary conditions.

FIGURE 6

shown, the system of differential equations for $\bar{p}$ and the equations of motion (1.) can be expressed in the canonical form given next. The Pontryagin Maximum Principle then states that H must be extremized with respect to the value of χ at each time along an optimum trajectory. For our example, the equations for $\dot{\overline{p}}$ are written out in (7.), and the conditions that H be maximized in (8.) and (9.). We are thus led to the two-point boundary value problem defined by equations (6.), (7.) and (1.) with end conditions (2.) and (3.) and the added restriction (8.). In fact, except for (8.) this is the same system as obtained by the classical approach, with "p's" substituted for "λ's". Also, (8.) can be derived from further necessary conditions in the classical approach. The advantages claimed by the Pontryagin approach are perhaps a cleaner analysis, and the capability of handling certain constraints and classes of admissible functions which have not been successfully treated by the classical method.

However, as mentioned before, a two-point boundary value problem remains. An analytic solution to this would be quite useful, but at this time most solutions are obtained by simply guessing a set of the remaining initial values and T, solving the resulting one-point boundary value problem, and iterating for the solution of the two-point problem.

E. GRADIENT APPROACH

Another approach to the variational problem is the Gradient method, in which the optimum χ function is approached through non-optimum functions at the same time that one or a sequence of two-point boundary value problems are solved by iterated one-point problems. This is done as follows (see Figure 7): A function space is set up in which each point corresponds to an entire function $\chi(t)$. The "distance" between two control functions in this space is defined in some manner, such as the essentially Euclidean metric shown in the figure. Then it can be shown that the path of steepest descent from an arbitrary "point" $\chi(t)$ to the optimum $\chi(t)$ has the tangent given next on the chart, which turns out to be the partial with respect to χ of the Pontryagin H function. The term "steepest", of course, means that it increases the quantity to be maximized as much as is possible by a change in $\chi(t)$. Thus, taking finite steps down this tangent or gradient vector would define the scheme shown for computing a sequence of improving χ functions, where the gradient vector is to be recomputed when it differs sufficiently from the actual path of steepest descent. It is clear, then, that when the optimum χ is reached, $\overline{\chi}^{k+1}$ would equal $\overline{\chi}^{k}$, and each component of H_χ would have a zero magnitude. Thus, the stepwise procedure brings a satisfaction of one necessary condition obtained in the Pontryagin approach. The derivatives of $\overline{p}$ to be computed are the same as given in the Pontryagin approach, the only difference being that they

GRADIENT APPROACH

Define distance in control function space,

e.g.: $$||\bar{\chi} - \bar{\chi}'|| = \sqrt{\sum_{1}^{r} \int_{0}^{T} (\chi_i - \chi_i')^2 \, d\tau}$$

Then the tangent to the direction of maximum increase in S at a "point" $\bar{\chi}$ in the function space is $\bar{v}$, where

$$v_i = -\bar{p} \cdot \bar{f}_{\chi_i} = \frac{\partial H}{\partial \chi_i}$$

To generate a sequence of improving control function $\bar{\chi}$, compute

$$\bar{\chi}^{(k+1)} = \bar{\chi}^{(k)} + \alpha \bar{v}$$

Our example has a one dimensional $\bar{\chi}$, and the sequence is given by

$$\chi^{(k+1)} = \chi^{(k)} + \alpha \left(p_1 \frac{F}{x_5} \cos \chi - p_2 \frac{F}{x_5} \sin \chi \right) \quad 9.$$

Solve the resulting one point boundary value problem: 1., 2., 3., 6., and 9. with arbitrary χ, and iterate.

FIGURE 7

are now computed along non-optimum trajectories for all but the last step. The advantages claimed by this approach are that certain problems involving discontinuous functions, such as between stage coasting periods, are more easily handled, and some of the difficulties encountered in the iterative solution of the two-point boundary value problems of the other approaches are avoided.

The decks now in use will be described briefly.

A. S-SERIES COMPUTER DECKS

In the S-Series, rotational dynamics are simplified to the steady state solution, and motion is assumed to be planar about a non-rotating spherical earth with or without atmosphere. The trajectory shape is determined by prescribing certain variables, such as angle of attack α or thrust acceleration direction χ, as definite functions of other variables. For reliability and speed of operation, specific decks are set up for fairly specific purposes; as an example, for atmospheric propelled flight performance studies, atmospheric reentry studies or control studies, all on simplified models of the physical situation. There are around eleven such decks in use, most of them using Runge-Kutta integration.

B. B-SERIES COMPUTER DECKS

A B-Series of decks is based on an accurate representation of all flight mechanical and rigid body dynamical details. Flight is about a rotating oblate spheroid with atmosphere. These decks are used to check results and design work of the S-Series decks, and to provide refined and accurate information necessary for the actual flights of vehicles.

C. V-SERIES COMPUTER DECKS

The V-Series decks are again primarily used for performance and guidance design work. The common feature of these decks is the replacement of the arbitrary shaping functions found in the S-Series with functions derived according to the calculus of variations. The first decks in this series were propelled flight in a vacuum, 2 dimensional, with a spherical earth assumed. These decks are written in both cartesian and polar form, with both Runge-Kutta and Taylor Series integration procedures. These decks have been involved in most of the recent upper stage performance and guidance study work.

The effect of atmosphere has been added with the assumption of small angles of attack to form another deck for propelled flight.

Another deck under experiment is using the flat earth solution as a base and using a series type integration to evaluate the perturbations due to the spherical earth.

F. DYNAMIC PROGRAMMING APPROACH

The last approach mentioned on the earlier chart is that of dynamic programming. This approach is based entirely on Bellman's Principle of Optimality that states that if a trajectory from t_o to T is optimal, then any sub arc of that trajectory from any intermediate time, t, to the final time must also be optimal. In the computation procedures then, to obtain the optimum trajectory from t_o to T, the interval is covered with a finite grid which may be traversed in a finite number of ways. Starting at T and working backwards on the grid, an analagous grid is set up which represents at each point the value of the quantity to be extremized if the optimum trajectory is traversed from the corresponding point on the original grid to the desired end conditions at T. Upon reaching t_o by this procedure an optimum trajectory is determined by following the path back through the first grid defined by the smallest values in the second grid. The advantages claimed by this approach are similar to those of the gradient approach, since it too is a stepwise numerical procedure. Rigorous analytic foundations for this approach have not been found to be readily accessible in the literature.

Certainly approaches other than these four exist, and more will be developed. At present, only the first three and the corresponding one-and two-point boundary value problems are being attacked by Future Projects Branch and the associated contractors, as indicated in Figure 2.

SECTION III. APPLICATIONS

The results of such efforts usually are seen in the form of the various "deck's" for trajectory computation used by the branch for application in the various fields shown in Figure 2. The "deck" itself will be considered to be the systematic carrying out of a given sequence of computations, usually the solution of a one-point boundary value problem. In order for the decks to be used effectively, iterative schemes must also be programmed for use with the decks. Each deck may require a particular type of iterative process, so that having a deck available does not always mean that production can be run on it. The problem involved is that of moving from an arbitrary point on an implicitly defined surface to a desired point. Various procedures for doing this are in use or being developed, including first and second order differential correction methods, and the ordinary gradient method.

The development of a 3 dimensional variational deck has been performed by the Auburn University contractors. An iteration scheme for use with that deck is now under development in the branch.

A low-thrust deck is presently being developed by Grumman Aircraft Engineering Corporation, by the gradient approach, in both two and three dimensions for application to interplanetary and near earth orbit transfer.

Another deck being experimented with in-house is a calculus of variations reentry deck which minimizes the integral of the square of the total drag. The results of a trial run are shown in Figure 8.

EXAMPLE OF AN APOLLO REENTRY TRAJECTORY
RUN ON THE EXPERIMENTAL 2-DIM CALCULUS OF VARIATIONS REENTRY DECK

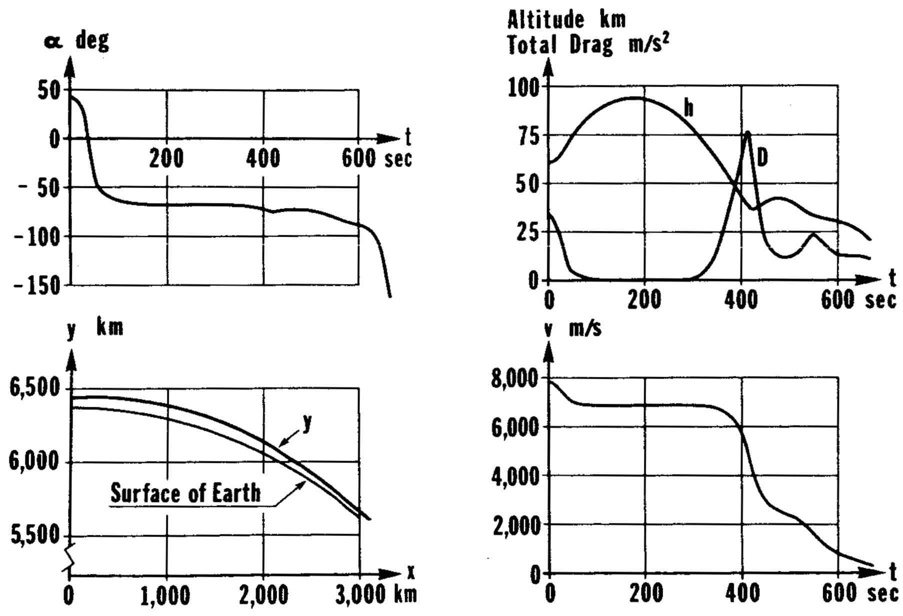

FIGURE 8

METHODS OF EXPLOITATION OF LARGE-SCALE AUTOMATIC COMPUTERS

By

Nolan J. Braud

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METHODS OF EXPLOITATION OF LARGE-SCALE AUTOMATIC COMPUTERS

By

Nolan J. Braud

SECTION I. INTRODUCTION

Some of the problems encountered in studies of Space Flight and Guidance Theory are quite extensive and require a judicious utilization of large scale computers and computer programs. The purpose of this report is to present some of the scientific disciplines involved in achieving numerical results for such problems.

Three major problem areas are considered for Large Computer Exploitation. They are a Function Differential Generator, the development of a Statistical Model and Investigations of Multivariant Functional Models (see Figure 1). The areas of manpower utilization under the contract "Space Flight and Guidance Theory" are also indicated, where the symbols representing various contractors are defined on page 83 of this report.

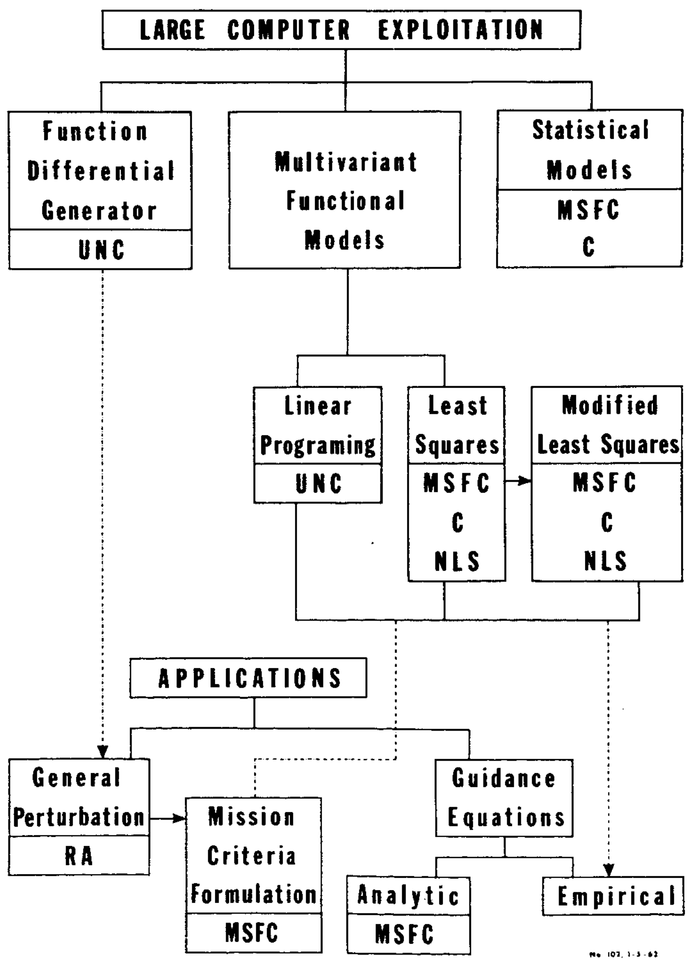

FIGURE 1

SECTION II. DISCUSSIONS

A. FUNCTION DIFFERENTIAL GENERATOR

The Function Differential Generator is a computer program that will differentiate a certain class of algebraic and transcendental expressions automatically. The program utilizes a set of algorithms which transforms a given expression in such a way that its derivatives are obtained by successive application of the basic rules of differentiation for elementary functions. It is interesting to note that the allowable class of expressions must be closed under the operation of differentiation.

The work on the Function Differential Generator is being done exclusively by the University of North Carolina. Their procedures entitled, "Analytic Differentiation by Computer," are described in MTP-AERO-61-91 dated December 18, 1961.

The program operates in the following fashion after having designated the expression to be differentiated and having chosen the variables of differentiation. The first step is to transform the given expression into a correlated set of triples (operand-operator-operand) such as $A \times B$. This results in a matrix representation of the expression in a parenthesis-free form. The next step is to determine the derivative of each of the triples or rows of the matrix. Then follows a collection, in a correlated fashion, of these elementary derivatives. Hence, the derivatives of an expression are acquired by successive application of the basic rules of differentiation for elementary functions.

The work on this program is not complete at this time, but a preliminary application of the procedures has been made to determine the coefficients for the Taylor's Series expressions of the simplified flat-earth calculus of variations problem. Only first order derivatives were evaluated; however, the results would indicate a promising future for a computer program of this nature.

B. STATISTICAL MODEL DEVELOPMENT

The development of Statistical Models and the investigation of Multivariant Functional Models arose out of requirements in the implementation of the Path-Adaptive Guidance Mode. These two areas of discipline are being used directly in the writing of the guidance functions of the adaptive mode. The nature of the steering and cutoff function in the Path-Adaptive Guidance Mode are shown in Figure 2, where the "o" subscript refers to instantaneous conditions.

NATURE OF THE STEERING AND CUTOFF FUNCTION IN THE PATH-ADAPTIVE GUIDANCE MODE

STEERING

$$\chi_0 = \chi_0\left[x_0,\ y_0,\ \dot{x}_0,\ \dot{y}_0,\ \left(\frac{F}{m}\right)_0, \left(\frac{\dot{m}}{m}\right)_0,\ t_0\right]$$

CUTOFF

$$t_f = t_f\left[x_0,\ y_0,\ \dot{x}_0,\ \dot{y}_0,\ \left(\frac{F}{m}\right)_0, \left(\frac{\dot{m}}{m}\right)_0,\ t_0\right]$$

FIGURE 2

A Statistical Model is understood to be a collection of quantative data that represent a physical situation. The generation of a Statistical Model that the guidance equations are to represent involves a fairly extensive performance and trajectory investigation. The study involves the development of a manifold of trajectories that represents all possible disturbances which can affect vehicles of the Saturn class and allow for mission achievement. The trajectories are all determined by applying the theory of the calculus of variations, which results in the minimization of fuel consumed for each case. Trajectories determined under such principles result in the specifying of an optimum value for the steering and cutoff parameters at each point along their path. By considering the volume of trajectories as a collection of such points, we arrive at the tabulated values which are to be approximated. The development of Statistical Models that represent the quantitative data for the Adaptive Guidance Mode is done by MSFC, with some assistance from Chrysler Corporation.

C. MULTIVARIANT FUNCTIONAL MODELS

The studies in the area of Multivariant Functional Models are concerned with the investigation of various curve fitting procedures to determine which is preferred for our needs in guidance function writing. At the present time a polynomial form has been tentatively selected for the representation of the guidance functions. The expansion of the series used are indicated in Figure 3. From this it is seen that the instantaneous steering and cutoff functions are expressed in a series of terms in the state and performance variables. The coefficients (a_i) are constants that are to be approximated by multivariant expressions.

EXPANSION OF THE SERIES USED IN THE IMPLEMENTATION OF THE PATH-ADAPTIVE GUIDANCE MODE

$$\chi_0 \text{ or } t_f = a_0 + a_1 x + a_2 y + a_3 \dot{x} + a_4 \dot{y}$$

$$a_5 \left(\frac{F}{m}\right) + a_6\left(\frac{\dot{m}}{m}\right) + a_7 t + a_8 x^2 + a_9 xy$$

$$a_{10} y^2 + \ldots + a_{16} \dot{x}\dot{y} + \ldots + a_{18} x\left(\frac{F}{m}\right) + \ldots$$

$$a_{33}\left(\frac{F}{m}\right)\left(\frac{\dot{m}}{m}\right) + \ldots + a_{35} t^2 + \ldots$$

FIGURE 3

The Multivariant Functional Models which have received most emphasis are Least Squares, Modified Least Squares and Linear Programming (see Figure 4). These methods are being compared for economy, accuracy, speed and ease of use.

MINIMIZING CRITERIA FOR CURVE FITTING

LINEAR PROGRAMING

$$L_1 ; \quad \sum_{K=1}^{n} \left| P(\bar{z}_K) - f(\bar{z}_K) \right|$$

$$L_\infty ; \quad \max_{K=1,\ldots,n,} \left| P(\bar{z}_K) - f(\bar{z}_K) \right|$$

LEAST SQUARES

$$\sum_{K=1}^{n} \left[P(\bar{z}_K) - f(z_K) \right]^2$$

FIGURE 4

Linear Programming is a mathematical technique for finding an optimum solution to sets of simultaneous equations with more variables than equations, where the variables cannot have negative values. Polynomial approximation problems such as those encountered in the empirical writing of the guidance functions can be formulated for solution by the Linear Programming methods. The problem must first be stated as either a maximization or a minimization problem, and in the form of an inequality. Then slack variables are introduced which transform the problem statement into sets of equations that are solved by some conventional means of solving simultaneous equations. The University of North Carolina is conducting the investigations on Linear Programming application to the problem of writing guidance equations.

Least Squares approximations are achieved under the principle that the best value of a quantity that can be deduced from a set of observations is that for which the sum of the squares of the deviations from the observed is a minimum.

Figure 4 displays the minimizing criteria that have been used for curve fitting purposes. The L_1 criteria requires that the sum of the absolute deviations be a minimum, whereas the L_∞ criteria requires that the absolute value of the maximum deviation be minimized. The minimizing criteria under the least squares principle is that the sum of the squares of the deviations should be a minimum.

SECTION III. APPLICATIONS

These criteria have been used to generate many guidance polynomials. The most favorable results have been generated by the method of least squares. This stems from the fact that much more effort has been devoted to the least squares methods than to the other techniques. Even though linear programming has not been thoroughly investigated, it seems to offer distinct advantages in the area of fitting rational polynomials, where more accuracy is anticipated as well as a reduction in the number of terms in the polynomials. This approach to the problem will be undertaken by Chrysler Corporation in the near future. Another linear programming technique which may offer some advantage is a method available for approximating a polynomial of a given form which requires that the absolute deviation at the K-th point be less than a preassigned value. If the problem cannot be solved under the specified constraints, the linear programming routine will indicate that no solution can be obtained.

The prime consideration in the area of modified least squares is that of fitting the residuals. By this method a polynomial of a given form is fitted by the conventional least squares approach. Then succeeding or higher order terms are fitted to the residuals or errors of the original polynomial. By approaching the curve fitting problem in this fashion, advantage is gained by the fact that a smaller set of simultaneous normal equations are solved in each step of the problem. Hence, greater accuracy is maintained in the over-all solution to the problem.

SECTION IV. CONCLUSIONS

There are many problem areas, common to almost any multivariant approximating procedure, that need to be investigated before a firm control will be had over the problem of writing guidance functions. Some of these problem areas are:

1. The investigation of existence and uniqueness conditions for a function that will represent the guidance functions while minimizing the deviations.

2. The determination of a means of selecting the optimum data sampling from a large statistical model.

3. The problem of weighting the data sampling.

4. The use of probability theory.

5. The use of mathematical statistics.

6. The choice of an optimum set of parameters to be represented in the polynomial.

7. The means of achieving an optimum solution to a large number of simultaneous equations.

It is generally believed that a stronger control over these areas is needed in order to obtain the optimum solution to our problems.

PRESENT THEORY AND TECHNIQUES APPLIED TO SPACE VEHICLE PROBLEMS

by

John B. Winch

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PRESENT THEORY AND TECHNIQUES APPLIED TO SPACE VEHICLE PROBLEMS

By

John B. Winch

SECTION I. INTRODUCTION

The preceding chapters in this paper have presented the scientific disciplines being studied by Future Projects Branch. The purpose of this section is to illustrate how these disciplines are being applied as needed by the branch in the solution of flight mechanical problems. First, the performance and guidance problems and the bringing together or integration of the total vehicle system will be discussed. These problems will then be further illustrated by applications to two simple problems pertaining to missions which have been assigned to the Saturn C-1.

The first application is that of a range independent injection into a circular orbit. The second is a range independent reentry mission. These problems are solved by identical procedures so that comparisons can be made. Some results are given, but they are not intended to represent any best or final solutions.

SECTION II. DISCUSSIONS

The term "System Integration" is used here to cover the problems involved in bringing the various parts of a vehicle system, which have been studied separately, together. Independently, the parts have their own optimum solutions, but these solutions depend also upon the state of the remaining parts of the system, so that an over-all optimization problem exists. Some of the major parts of the system are: propulsion and propellant distribution between stages, structures, instrumentation, computer, aerodynamic characteristics, control, guidance, and trajectory shape.

The system integration must be studied as affected by such things as mission, and removable constraints.

The term "mission" is defined here to mean the results desired of the vehicle flight. Examples are: an impact at a prescribed point on the lunar surface, a rendezvous with a body in some prescribed orbit, or a simulation of reentry from a lunar flight. "Removable constraints" are defined as those constraints which can be either modified or removed completely. For example: an altitude limitation due to aerodynamic heating considerations which can be removed by adding heat protection material to the vehicle, flight path limitations for tracking which can be removed by adding more tracking stations, or flight time to the moon for manned flight which can be removed by adding more life support equipment for the astronaut. Other constraints were mentioned in the chapter on "Calculus of Variations."

The missions are normally defined at higher levels of the administration, and the removable constraints depend on hardware developed in other branches and divisions. The missions requested and the development of hardware depend upon an early feedback from the system integration studies. Later work refines the system so that the best possible satisfaction of the mission is obtained with the developed hardware.

The evaluation of a given hardware configuration as applied to a given mission is generally made in two phases - performance, and guidance.

To illustrate these problems, we will consider as the given mission a lunar impact. In solving a performance problem, it is conceptually easier to start at the point of mission fulfillment and work backward. The first step then would be to generate a family of optimum trajectories going from the moon back to the earth. The generation of the trajectories could be accomplished through the use of the computational techniques discussed in the chapter on "Celestial Mechanics - Special Perturbations." These free flight trajectories would form an envelope around the earth.

The next segment of the problem would involve the generation of a family of optimum powered flight trajectories from the launch site to intersection with the family of lunar trajectories. These powered flight trajectories might include parking orbits and/or orbital rendezvous as dictated by the vehicle and system constraints on the problem. The method of generation of the optimum powered flight trajectories would be the calculus of variations procedures discussed in the chapter on "Calculus of Variations." The calculus of variations technique to be used would be selected on the basis of the one which was most easily adaptable to the problem to be worked.

The definition of the mathematical surface joining the powered flight trajectories to the lunar trajectories is a most difficult problem, and represents an area in which a large effort would be required. Let it suffice to point out that the cutoff point of powered flight must consider both propelled flight optimization and the optimization of the midcourse corrections to follow. A tradeoff situation must be resolved between the two parts of the problem so that the entire system is optimized, not just one segment of it.

The guidance problem may be more easily understood by starting from lift-off and following the flight chronologically. Guidance during propelled flight could be provided by the adaptive guidance mode, which involves the use of the numerical techniques discussed in the chapter on "Large Computer Exploitation." In principle, the techniques discussed could also be used in representing a lunar trajectory mission criterion; however, it would probably be better to make use of the schemes discussed in the chapter on "Celestial Mechanics - General Perturbations."

The technologies used in these studies are still in a state of development. This necessitates studies into an apparent problem area before a solution to the problem is formally requested. For this reason, studies have been initiated in the field of trajectory studies for low thrust vehicles, such as would be the case if ion propulsion were to be used for a stage. Variable thrust is another physically possible innovation which may find practical use in a few years. Reentry problems are present now which demand that some studies be conducted on optimum flight paths during reentry into the atmosphere, since these determine the conditions that must be met by the preceeding phases of flight.

The one sample problem of lunar impact is not the only example where all areas of scientific investigation discussed in this paper would be utilized. A brief inspection of future missions for Saturn vehicles shows flights of the following types:

1. Earth-Orbit
2. Lunar impact
3. Lunar circumnavigation
4. Soft lunar landing
5. Interplanetary

Each of the missions listed would demand input from all of the areas of scientific endeavor in order that a well founded scientific analysis of the problems could be conducted.

SECTION III. APPLICATIONS

Two problems were chosen for applications. These will be designated Case I and Case II. The guidance application was made using the same terms and the same pattern of selecting points for each part of both cases. This was done to bring out the effects of changes in mission criteria formulation. It is planned to bring out a note covering more details at a later date. This later note will present optimum fits as to the selection of terms and points.

A. THRUST AND WEIGHT DATA

The Saturn C-1, Block II vehicles are two stage configurations. The first stage (S-I) is powered by eight 188K lb engines, four of which are gimbal mounted for vector thrust control. The thrust and weight characteristics and the mass distribution data were obtained from the Saturn Design Criteria Book dated May 12, 1961.

B. CASE I

For this case the mission criteria was assumed to be that of injection into a circular orbit. No requirement was assumed for the plane of the orbit or the position of the vehicle in the orbit. This is referred to as range independent in the sense of trajectory optimization. The circular orbits desired were assumed to be one, two, and three hundred nautical miles above the surface of the earth.

The performance problem is solved first. In doing this, the first stage trajectories were shaped so as to favor a seven engine acceleration history from lift-off. This constraint was imposed by rigid body control considerations. Subject to the constraints given, the criteria used to select a first stage tilt program was that the second stage, assuming idealized performance, deliver maximum cutoff weight into the specified injection conditions. The notation 8/7 and 7/7 will be used to designate trajectories for which the first stage has simulated a flight with 8 and 7 engines, respectively, following the tilt program just described. These two cases define the limits of the volume of first stage trajectories for practical purposes. The second stage flight of this two stage missile is above the atmosphere. Therefore, any desired angle of attack can be obtained without undue aerodynamic forces restraining the missile. Full guidance is used during the second stage in all cases. This sample problem was computed considering in-plane flight only. The same techniques used for this problem may be used to solve out-of-plane cases.

In solving the guidance problem, the steering equations were developed by empirical methods. This development was started after the performance problem had been solved. The first step in the empirical method is to establish a statistical model. Reference is made to the section on "Large Computer Exploitation." The statistical model was established by computing a family of optimum trajectories which satisfy the mission criteria. This family of trajectories covered the volume of space through which any vehicle might be expected to fly, up to a given probability of occurrence. The "points" (to be defined below) of these trajectories make up the statistical model.

The trajectories were computed using a two dimensional space-fixed cartesian coordinate system. The origin was chosen as the center of the earth. The positive y-axis passes through the launch site. The x-axis is in the plane of flight and is positive down range. Both position and velocity measurements were assumed to be available, transformed into this system. The term "point" (as used in the statistical model) is defined as the parameters which affect the desired thrust angle for an optimum trajectory. These parameters are position ($\overline{x}$), velocity ($\dot{\overline{x}}$), and engine performance data (F/m, $\dot{m}/m$, etc.). F/m is the ratio of thrust to mass. $\dot{m}/m$ is the ratio of flow rate to mass. The thrust angle (χ) is defined as the angle from the y-axis to the long axis of the missile. The desired tilt angle (χ) is assumed to be a function of the point in phase space.

Thus,

$$\chi = \chi\ (\bar{x},\ \dot{\bar{x}},\ F/m,\ \dot{m}/m,\ t)\ .$$

The statistical model then is comprised of points of the space and their related χ value.

The statistical model is composed of an arbitrary sampling of points from the phase space. This sampling was made by selecting points on 32 trajectories at 15 second intervals from 135 to 585 seconds and at 5 second intervals from 585 seconds to cutoff. The thirty-two trajectories were selected in the following manner. Trajectories for the first stage were run for two cases. These cases were 7/7, and 8/7. Two points were interpolated between these extremes. Second stage trajectories were computed from these four initial points, with the following second stage conditions: first, standard (or expected) second stage performance; second, variations in thrust level of ± 2%; and third, variations in specific impulse of ± 4 seconds. This established a total of twenty trajectories. Twelve additional second stage trajectories were computed with standard performance and initial state coordinates varied between the 8/7 and 7/7 end points. This sampling resulted in a total of about 1220 points of the phase space. This procedure was used throughout the study for comparison purposes.

A few typical polynomials were arbitrarily selected for this presentation. These polynomials contain 7, 8, 28, 36, 47, and 57 terms. The seven term polynomial contained only first order terms without $\dot{m}/m$; the eight, all first order terms; and the 47 and 57, selected first, second, and third order terms. Again, the same terms were used for all parts of this study. It should be noted that this selection is not the best. All possible steering polynomials should be considered. The computer capacity dictates that terms higher than third order may not be used. There are 120 terms up to and including third order, and 10^{36} possible combinations of these terms. Although many of these possibilities may be eliminated by inspection, the problem of selecting a best polynomial for a specified mission is still a tremendous task.

The measure of error of the steering function is hard to determine. Errors result because the statistical sample only gives a discrete representation of the phase space, and also cannot be fit exactly. Two effects of errors in the steering function may be noted. The first is that the mission criteria cannot be met exactly (except for a mission dependent upon one function which increases monotonically with time). The second is, that more than the theoretical minimum of propellants is consumed. These effects may be measured on each trajectory computed under simulated active guidance. Each set of assumptions for a simulation results in a corresponding value for mission achievement error and additional propellants consumed. By choosing several sets of assumptions from the statistical model which was fit, and combining the resulting set of errors and additional propellants required, some measure of the error of fit for that particular steering function can be computed. In this application, only the limiting sets of assumptions (8/7 and 7/7) were used to check the steering functions in this way. Detailed analysis of errors was not considered economical for this application.

Another measure of accuracy of the steering function is the RMS (root-mean-square) error with respect to the points of the statistical model. This criteria is used mainly as a mathematical guide, since it depends upon the particular choice of points for the statistical model, and is more in the nature of a necessary rather than sufficient requirement. That is, a small RMS is necessary for a good steering function, but does not guarantee one that meets the practical requirements of the problem.

The two polynomials without ($\dot{m}/m$) terms were generated to investigate the effect of leaving out this parameter. There is some doubt as to the accuracy and reliability of the measurement of this term from the engineering point of view. No firm conclusion as to the effect of dropping this term may be deduced from this study. However, some idea as to the error caused may be inferred from these special cases. It may be mentioned that in range and time dependent cases $\dot{m}/m$ may play a more important role in the steering equations.

The cutoff equation used in this study caused cutoff to occur when the desired velocity was reached. This expression was used so that errors would appear in path angle and altitude only. Also, it enables a measure of the steering equation independent of the cutoff equation to be made. Later studies will integrate the errors caused by both cutoff and steering equations.

Tables 1 through 3 present the coefficients of the first order terms of the six steering polynomials discussed previously.

TABLE 1

SATURN C-1: 100 N.M. RANGE-INDEPENDENT MISSION

Number of Terms Used for Steering Function \ Coefficients of First Order Terms	a_0 Constant	a_1 x	a_2 y	a_3 $\dot{x}$	a_4 $\dot{y}$	a_5 F/m	a_6 t	a_7 $\dot{m}$/m	RMS Error
1. 8 Terms 1st Order	−1325	324.6	236.5	−47.79	29.05	3.268	−58.08	−90.24	2.31
2. 36 Terms 1st & 2nd Order	−6646	−6092	599.3	1585	618.6	80.52	870.6	9311	.64
3. 47 Terms 1st, 2nd & Selected 3rd	6758	−1140	−2188	19.33	130.4	−1070	1484	17791	.45
4. 57 Terms 1st, 2nd & Selected 3rd	13247	479.1	−3824	−377.6	−41.90	−10168	0	340307	.22
5. No. 1 w/o $\dot{m}$/m Terms	−1349	320.9	238.6	−42.29	30.35	.6217	−59.12	0	2.33
6. No. 2 w/o $\dot{m}$/m Terms	−6303	−4773	518.8	1669	728.1	248.9	632.4	0	.72

TABLE 2

SATURN C-1: 200 N.M. RANGE-INDEPENDENT MISSION

Coefficients of First Order Terms / Number of Terms Used for Steering Function	a_0 Constant	a_1 x	a_2 y	a_3 $\dot{x}$	a_4 $\dot{y}$	a_5 F/m	a_6 t	a_7 $\dot{m}/m$	RMS Error
1. 8 Terms 1st Order	−1256	285.3	217.9	−42.84	25.02	4.114	−52.74	−67.19	1.92
2. 36 Terms 1st & 2nd Order	2821	−4087	−1506	699.7	133.7	62.52	742.9	−2124	.43
3. 47 Terms 1st, 2nd & Selected 3rd	11953	−2672	−4109	25.87	86.20	−1589	1946	56332	.35
4. 57 Terms 1st, 2nd & Selected 3rd	5562	989.0	−1438	−305.0	−144.6	1736	0	−102790	.12
5. No.1 w/o $\dot{m}/m$ Terms	−1268	281.5	218.5	−39.45	26.15	2.287	−53.06	0	1.94
6. No.2 w/o $\dot{m}/m$ Terms	1518	−4256	−1291	846.0	287.0	118.2	673.1	0	.59

TABLE 3

SATURN C-1: 300 N.M. RANGE-INDEPENDENT MISSION

Coefficients of First Order Terms / Number of Terms Used for Steering Function	a_0 Constant	a_1 x	a_2 y	a_3 $\dot{x}$	a_4 $\dot{y}$	a_5 F/m	a_6 t	a_7 $\dot{m}/m$	RMS Error
1. 8 Terms 1st Order	− 1146	226.4	188.6	− 22.81	27.58	3.187	−46.89	−22.47	1.76
2. 36 Terms 1st & 2nd Order	11620	− 1114	− 3583	33.49	−387.2	−3.067	501.2	−6620	.25
3. 47 Terms 1st, 2nd & Selected 3rd	10681	− 2705	− 3578	13.10	10.33	−994.7	1462	36544	.21
4. 57 Terms 1st, 2nd & Selected 3rd	9458	839.5	− 2893	−208.1	−198.5	−334.1	0	1951	.16
5. No. 1 w/o $\dot{m}/m$ Terms	− 1146	224.8	188.4	−22.09	27.83	2.623	−46.80	0	1.76
6. No. 2 w/o $\dot{m}/m$ Terms	11689	− 2253	− 3820	223.3	−275.0	−51.26	609.2	0	.43

In the case of polynomials 2, 3, 4, and 6, the coefficients of the higher order terms are not shown. A general tendency which is displayed is that, for a given polynomial, the RMS error decreases as injection altitude is increased. This tendency is probably a result of the fact that the volume of initial conditions is decreased for increased injection altitude. This phenomenon results from the manner in which the limiting trajectories were generated, and is a physically realistic situation.

Trajectory simulations were made on the limits of the volume fitted. The results of these similations are presented in Tables 4 through 6.

TABLE 4

SATURN C-1: 100 N.M. RANGE-INDEPENDENT CIRCULAR MISSION

Number of Terms Used for Steering Function	Motors In Booster	Wt (lb) C of V	Wt (lb) χ Run	ΔW (lb)	$\Delta\vartheta$ (deg)	Δy (km)
1. 8 Terms 1st Order	8/7	35393	35558	165	.39	4.9
	7/7	35926	35822	– 104	–.67	–3.7
2. 36 Terms 1st & 2nd Order	8/7	35393	35438	45	.10	.2
	7/7	35926	35932	6	.01	–.3
3. 47 Terms 1st, 2nd & Selected 3rd Order	8/7	35393	35412	18	.04	.4
	7/7	35926	35921	– 5	0	.4
4. 57 Terms 1st, 2nd & Selected 3rd Order	8/7	35393	35418	25	.06	–.2
	7/7	35926	35936	10	.05	–.1
5. No. 1 without $\dot{m}/m$ Terms	8/7	35393	35564	–29	.40	5.1
	7/7	35926	35816	–110	–.68	–3.4
6. No. 2 without $\dot{m}/m$ Terms	8/7	35393	35428	35	.08	.5
	7/7	35926	35926	0	–.02	–.3

TABLE 5

SATURN C-1: 200 N.M. RANGE-INDEPENDENT CIRCULAR MISSION

Number of Terms Used for Steering Function	Motors In Booster	Wt (lb) C of V	Wt (lb) χ Run	ΔW (lb)	Δϑ (deg)	Δy (km)
1. 8 Terms 1st Order	8/7	32812	32843	31	.47	4.6
	7/7	32756	32817	61	−.69	−2.8
2. 36 Terms 1st & 2nd Order	8/7	32812	32815	3	.03	.4
	7/7	32756	32753	−3	0	.2
3. 47 Terms 1st, 2nd & Selected 3rd Order	8/7	32812	32812	0	.01	.4
	7/7	32756	32749	−7	−.01	.4
4. 57 Terms 1st, 2nd & Selected 3rd Order	8/7	32812	32816	4	.02	−.1
	7/7	32756	32756	0	.02	0
5. No. 1 without $\dot{m}/m$ Terms	8/7	32812	32845	33	.48	4.6
	7/7	32756	32814	58	−.69	−2.6
6. No. 2 without $\dot{m}/m$ Terms	8/7	32812	32817	5	.04	.4
	7/7	32756	32757	1	−.03	0

TABLE 6

SATURN C-1: 300 N.M. RANGE-INDEPENDENT CIRCULAR MISSION

Number of Terms Used for Steering Function	Motors In Booster	Wt (lb) C of V	Wt (lb) χ Run	ΔW (lb)	Δϑ (deg)	Δy (km)
1. 8 Terms 1st Order	8/7	29486	29335	−151	.62	4.9
	7/7	28567	28803	236	−.80	−2.6
2. 36 Terms 1st & 2nd Order	8/7	29486	29481	−5	.01	0.3
	7/7	28567	28558	−9	.01	0.3
3. 47 Terms 1st, 2nd & Selected 3rd Order	8/7	29486	29487	1	−.01	0.0
	7/7	28567	28569	2	−.02	0.1
4. 57 Terms 1st, 2nd & Selected 3rd Order	8/7	29486	29481	−5	.02	0.2
	7/7	28567	28566	−1	.00	0.0
5. No. 1 without $\dot{m}/m$ Terms	8/7	29486	29335	−151	.62	4.9
	7/7	28567	28801	234	−.80	−2.6
6. No. 2 without $\dot{m}/m$ Terms	8/7	29486	29479	−7	.03	0.3
	7/7	28567	26582	−5	.00	0.2

The deviations listed were obtained by comparison of the nominal injection conditions (altitude = 100, 200, and 300 nautical miles; local circular velocity; 90 degree path angle) with the trajectory simulations. Cutoff was assumed to be given at the nominal velocity level, so that no velocity error is present in any of the cases. All hardware was assumed to function perfectly. A comparison is made of the cutoff weight obtained by calculus of variations optimized trajectories from the same first stage end points and the trajectory simulations using polynomial steering functions. In some cases it is observed that the polynomial program delivered a higher weight to the reference velocity level than the calculus of variations trajectory. This gain is the result of the deviation in the other end conditions, due to the inaccuracies in the steering function. The calculus of variations program will always deliver a higher payload if identical end conditions are achieved.

C. CASE II

The second mission investigated was a reentry flight. This mission was assumed to be independent of range in the same sense defined previously. Constraints were imposed on the first stage flight in the same manner as discussed before; that is, the tilt program was shaped for seven engine performance. The S-IV (upper stage) cutoff point was constrained by requiring that cutoff occur at an altitude of 120 km and a path angle of 94 degrees. In order to assure the maximum reentry velocity achievable by the stage, fuel depletion was assumed in every case.

The results obtained from this study are shown in Tables 7 and 8.

In Table 7 it will be observed that the RMS error goes up to 0.45 degrees for the 57 term polynomial as compared with 0.32 degrees for the 47 term polynomial and the fact that the selection of terms in the polynomial can be more important than the number of terms. It also shows that a given choice of terms for the polynomial may be good for one mission and not good for another. The 57 term polynomial was selected on the basis of its performance for the orbital missions.

TABLE 7

SATURN C-1: REENTRY TEST FLIGHT

Number of Terms Used for Steering Function \ Coefficients of First Order Terms	a_0 Constant	a_1 x	a_2 y	a_3 $\dot{x}$	a_4 $\dot{y}$	a_5 F/m	a_6 t	a_7 $\dot{m}/m$	RMS Error
1. 8 Terms 1st Order	-1107	305.6	201.3	-46.67	23.23	4.063	-53.51	-64.13	1.42
2. 36 Terms 1st & 2nd Order	3046	-239.8	-917.1	-781.3	-60.60	223.1	601.1	-13005	.38
3. 47 Terms 1st, 2nd & Selected 3rd	-656.2	-1765	-45.98	-70.58	68.60	628.3	-754.7	-24483	.32
4. 57 Terms 1st, 2nd & Selected 3rd	-21893	-329.5	6390	-422.8	-31.95	4063	0	-23184	.45
5. No. 1 w/o $\dot{m}/m$ Terms	-1139	305.8	205.5	-42.72	23.61	2.235	-56.08	0	1.45
6. No. 2 w/o $\dot{m}/m$ Terms	-921.0	616.4	304.9	-273.1	-115.4	-110.9	-199.2	0	.51

TABLE 8

SATURN C-1: REENTRY TEST FLIGHT

Number of Terms Used for Steering Function	Motors In Booster	Δ Alt (km)	ΔV (m/sec)	$\Delta\vartheta$ (deg)
1. 8 Terms 1st Order	8/7	4.46	6.74	.13
	7/7	-1.56	7.13	-.36
2. 36 Terms 1st & 2nd Order	8/7	-.22	5.82	.01
	7/7	-.39	6.01	.01
3. 47 Terms 1st, 2nd & Selected 3rd Order	8/7	-.65	5.84	.01
	7/7	-.81	6.82	.00
4. 57 Terms 1st, 2nd & Selected 3rd Order	8/7	-1.36	6.78	.03
	7/7	1.08	3.00	.04
5. No. 1 without $\dot{m}/m$ Terms	8/7	4.41	7.56	.16
	7/7	-1.21	6.42	-.36
6. No. 2 without $\dot{m}/m$ Terms	8/7	-.10	5.21	-.01
	7/7	-.17	5.61	-.04

SECTION IV. CONCLUSIONS

The results presented demonstrate the flexibility of the adaptive guidance mode. By using a sufficient number of terms in the steering polynomial, any reasonable accuracy of injection may be obtained, as required by the mission criteria. Conversely, if the accuracy which is desired for a given mission is not great, a polynomial having only a small number of terms can be developed which will produce injection within the tolerances given. In both cases the accuracy of the theoretical guidance mode may be clearly separated from hardware considerations, both in terms of loss of optimization and inaccuracy of mission fulfillment. Thus, the effect of any limitations or approximations can be studied.

Later publications will include the results of studies using more refined polynomials and techniques. Also more advanced and complex missions, such as orbital rendezvous, will be analyzed.

COORDINATION OF IN-HOUSE AND CONTRACTOR EFFORTS IN THE DEVELOPMENT OF GUIDANCE AND SPACE FLIGHT THEORY AND TECHNIQUES

by

David Schmieder

TABLE OF CONTENTS

COORDINATION OF IN-HOUSE AND CONTRACTOR EFFORTS IN THE DEVELOPMENT OF GUIDANCE AND SPACE FLIGHT THEORY AND TECHNIQUES

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David Schmieder

SECTION I. INTRODUCTION

We may summarize the work of Future Projects Branch by noting, (1) the limitations on the procedures now in common use, (2) what work is now being done and is planned to be done to remove such limitations, and (3) how it is planned to accomplish this through the coordination of in-house and contractor activities under the contract, "Guidance and Space Flight Theory."

SECTION II. DISCUSSIONS

A. LIMITATIONS ON PRESENT PROCEDURES

Perhaps the most frequently solved problem is the one-point boundary value problem. Performance surveys, control studies, and guidance error analyses require the solution of this problem; and it is also used in the iterative solution of the two-point boundary value problem. These solutions usually involve stepwise integration procedures which are limited in accuracy by truncation or round-off errors and in economy of operation by over pessimistic error approximations.

The solution of the two-point boundary value problems encountered in our work is limited almost entirely to the iterative methods using one-point solutions. Thus, in addition to having the limitations mentioned for the one-point problem, we have the difficulties associated with numerically finding the inverse of a function at an implicitly defined point. Also, it is difficult to know definitely when all solutions of interest have been found.

The computer time associated with such solutions is important for economy reasons in preliminary design work on the ground. The importance of time as a limitation of the method increases when the onboard computations for the adaptive guidance mode are considered.

The space trajectory computation procedures now in general use are "special perturbation" solutions and thus are expressed as one-point boundary value problems. Thus, within a certain accuracy, the space trajectory resulting from a given cutoff point can be computed, and the cutoff points necessary to achieve desired space trajectories can be iterated for; but the mission criteria formulation needed for the adaptive guidance mode is not supplied.

Since the methods mentioned so far are mostly numerical, they also carry the disadvantage of requiring a lot of experience by the user in order to decrease the man-hour time for solutions. This makes it difficult to obtain fast results that are sometimes needed as, for example, when changes in hardware characteristics are being considered.

In addition to these limitations, it has been found to be more effective to write "decks" for machine computation specifically for problems as needed for the development of tools and for the application to various missions as they come down from administration, usually with an associated time schedule. Thus, we are always limited to the state of approximation to the physical model that exists in decks presently checked out. In order to push back these limitations as needed, the following projects are being carried on in-house.

B. IN-HOUSE PROJECTS

Special Perturbation techniques for improving the one-point boundary value solution for space trajectories have been under development for some time and this work will continue. Also to be continued are: studies toward a non-iterative type solution to the two-point boundary value problem that results from the classical calculus of variations; the functional approximation of tabulated multivariable functions, as used to represent a volume of two-point boundary value solutions derived iteratively; and a general perturbation solution to space trajectories falling under the restricted three-body problem. The latter is being done with the cooperation of Dr. Schulz-Arenstorff of Computation Division.

Our work to date with the calculus of variations has been based mainly on the Euler-Lagrange necessary conditions. Investigations are being started toward an examination and application of sufficiency conditions found in the theory.

The number of iterative solutions of two-point boundary value problems to be made has been increasing continually. Thus, a study has been initiated to obtain quicker, better controlled, and more universally applicable routines for iterating solutions to this problem.

Existing calculus of variations decks are being extended to include some higher ordered perturbation terms, such as the gravity associated with an oblate earth, and atmosphere. Also, experimental calculus of variations decks are being set up for use in reentry studies.

These in-house studies are complemented by many studies which have been contracted out to industrial and university groups.

C. OUT-OF-HOUSE PROJECTS

In the calculus of variations, Auburn University has written a three dimensional deck and, together with General Electric of Philadelphia, will add further forces and study the optimization of dive trajectories with various types of control as applicable to the present Apollo concept. Vanderbilt University is beginning a theoretical study of transversality conditions for discontinuous arcs and the sufficiency conditions of the classical approach. Grumman is developing a low-thrust deck in two and three dimensions for planetary and near earth orbit transfer. The gradient approach to the variational calculus is applied. It may be noted that although some problems of the iterative two-point boundary problem are eased by that method, the disadvantage of a necessary experience factor is still present.

The general perturbation solution to the three-body problem, for application to lunar flights, is being attacked from different points of view by General Electric and the University of Kentucky.

The approximation of a tabulated function of several variables by means of a formal function easily evaluated on an onboard computer is receiving attention by Chrysler Corporation Missile Division, the University of North Carolina, and Northeastern Louisiana State College. North Carolina is also developing an automatic function differentiator for possible use in all of the problems.

Several studies are hoped to be initiated soon in connection with orbital rendezvous. Survey-type studies of orbital transfers with high thrust engines are planned for Grumman and United Aircraft. United Aircraft, General Electric, and Raytheon would study the return from orbital launch complex and Bendix would study the docking maneuvers. These surveys would be designed to show the effects of hardware restrictions, over-all capabilities, and the over-all optimum solutions.

It is planned for Grumman to develop steering equations for a lunar mission, and Chrysler for an earth satellite mission.

We do not have the manpower to attack all of these problems in-house, so that for timely solutions and a healthy relationship with a representation of the industrial and university capacity of the nation, we feel the contractual approach to these problems will prove to be a useful one.

D. COORDINATION OF IN-HOUSE AND OUT-OF-HOUSE ACTIVITIES

In order to coordinate the in-house and out-of-house activities, bi-monthly meetings have been set up as information exchange points. At these meetings both problems and solutions are presented for comment and criticism by all members. Smaller groups meet for more detailed exchange of information in the various special fields.

This arrangement produces an in-house load of keeping control over the contractors, their production and plans for future work. To meet this problem, certain members of Future Projects Branch have been designated as specialists in a given field. These specialists keep up with proposals in the field along with members of the contract group. The present organization is as given in Figure 1.

We rely on Dr. Sperling for all Special Perturbation Theory, decks, and advice. Mr. W. B. Tucker, with one mathematician, works with him. Dr. Schulz-Arenstorff of Computation Division is also relied upon for advice.

We have encouraged direct exchange of views and results between the various contractors. Our only requirement has been that we be informed by copy of the letters and information transmitted.

CONTRACT SUPERVISION
GUIDANCE AND SPACE FLIGHT THEORY
(OVERALL TECHNICAL SUPERVISOR, W. E. MINER)

FIELD	CONTRACTORS	FUTURE PROJECTS SPECIALIST
Large Computer Exploitation	C, UNC, NLS, (RTN)	N. Braud
Celestial Mechanics (General Perturbations)	UKY, GE	M. Davidson
Calculus of Variations Theory and Implications	VAN, AUB, NV, (MHW)	R. Silber
Survey Studies	BX, GQ, UR	B. Tucker
Reentry and Direct Calculus of Variations	GQ, GE, RTN	J. Winch

FIGURE 1
(See page 83 for definition of symbols)

New unsolicited bids concerning this contract are being received very frequently now, and create quite a job of evaluation. However, it is attempted to keep the door open for all proposals and criticisms by the contractors and new bidders. Over-all action may then be based on the criticisms of the contractors and the advice of the specialists.

SYMBOLS

MSFC	Marshall Space Flight Center
UKY	University of Kentucky
UNC	University of North Carolina
AUB	Auburn University
VAN	Vanderbilt University
NLS	Northeastern Louisiana State College
GQ	Grumman Aircraft Engineering Corporation
C	Chrysler Corporation
GE	General Electric Corporation
RA	Republic Aviation Corporation
MHW	Minneapolis-Honeywell Regulator Company
RTN	Raytheon Manufacturing Company
UR	United Aircraft Corporation
BX	Bendix Aviation Corporation
NV	North American Aviation, Inc.

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APPROVAL

MTP-AERO-62-21

STATUS REPORT #1

on

THEORY OF SPACE FLIGHT AND ADAPTIVE GUIDANCE

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